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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL MEMORANDUM 1259

THEORETICAL INVESTIGATIONS ON THE EFFICIENCY AND THE
CONDITIONS FOR THE REALIZATION OF JET ENGINES

By Maurice Roy

Translation of "Recherches Théoriques sur le rendement et les
conditions de réalisation des Systèmes Motopropulseurs a Réaction."
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PRELIMINARY NOTES ON THE EFFICIENCY OF PROPULSION SYSTEMS

Definitions - Notations

1. The concept of efficiency of propulsion in a fluid.

The so-called "propulsion system" is a system that transforms part of the available energy of a fuel into propulsive energy without directly participating in the support of the propelled system.

Such a propulsion system may be regarded as distinct from the propelled system, as is the case, for example, on the propeller-engine unit forming an engine nacelle in a dirigible or similar systems called power eggs (or engine nacelles) in certain types of airplanes where these nacelles are distinct from the body and the wing unit.

In these conditions, it is seen that the useful effect of this propulsion system is, as to force, the thrust which the said system can transmit to the propelled system. This thrust T is reckoned in the direction of the forward speed V and is taken positive in direction of this speed. As to power, the useful effect of propulsion can be represented by the work or energy (TV) per unit time of the thrust T , moving at speed V . The product TV is the useful power of the propulsion system.

This useful effect must be compared with the consumption of the available energy supplied to the system in unit time, which is obviously represented by the available energy of the mass of fuel consumed in unit time.

*"Recherches Théoriques sur le rendement et les conditions de réalisation des Systèmes Motopropulseurs à Réaction." Publications Scientifiques et Techniques du Ministère de l'Air, Service des Recherches de l'Aéronautique, 1930.

To find this available energy, it is necessary to resort to thermodynamics.

Without going into details outside the scope of the subject, it should be remembered that, in a system subjected to chemical transformation, such as the powder in a cannon or the fuel-air mixture in a heat engine, the available energy depends, in particular, on outside conditions. On the other hand, it is known that, even when these conditions are suitably specified, it is not possible, in general, to accurately determine the variation in the available energy that corresponds to the chemical reaction involved.

However, as a general rule, this available energy or, more precisely, its diminution between initial state and final state of the visualized reaction can be represented by its "calorific value" (also called heat value) of the system subjected to the chemical reaction in question.

This is, moreover, a convention universally employed in the study of heat engines and which is ordinarily adopted without even noting the theoretical objection which it raises.

In the present report, the energy available per unit mass of fuel is represented by its low heat value (that is, without condensed water) at constant pressure measured at standard atmospheric conditions (pressure: 760 mm of mercury; temperature: 15° C.). As this value, designated by L , is practically independent of the pressure and temperature changes of the surrounding air, the convention thus acknowledged will be valid whatever the altitude of operation.

Thus when m represents the mass of fuel consumed in unit time, the input of this propulsion system in unit time is equal to mL .

The efficiency of the system is, theoretically, the ratio of the useful effect to the input.

With the quantities T , V , m , and L being assumedly measured in consistent units,¹ the over-all efficiency will be defined by

$$\eta_g = \frac{TV}{mL} \quad (1)$$

¹M.K.S. units, where T is expressed in kg, V in m/sec., m in units of mass (weight in kg divided by 9.81) and L in kg per unit of mass.

It should be noted that this over-all efficiency as defined is not necessarily lower than or equal to unity, as it should be in order to completely correspond to the logical notion of efficiency.

2. Definition of thermal efficiency.

In the propulsion system, which is none other than a heat engine whose useful energy is realized in the form of energy of propulsion, it is necessary to distinguish between: (a) the active bodies, that is, those which, at the end of a period of operation in the cycle, are in a different physical or chemical state from their initial state; (b) intermediary bodies, that is, those whose state is the same at the beginning and at the end of a cycle. Among the latter category belong the solid elements of the system, as well as the fluid, which work in closed circuits (such as the cooling water or the lubricating oil, for example).

Among the active bodies belong the fuel, and generally, the air, whether the latter functions in the system as propellant or merely serves to dilute the products of combustion.

At entry into the system, these active bodies are in a certain initial state or state of admission (a). Both the air and the fuel are assumed to be at ambient pressure p_a and atmospheric temperature T_a . The fuel, in this state, moves with practically negligible speed, but the air may have an appreciable speed.

At the exit from the system, the active bodies find themselves in a certain final state or state of evacuation (e). In this state, the pressure is assumed to be uniform and always equal to the ambient pressure p_a . (Actually it can differ substantially when exhausting in a low-pressure zone, but this has no appreciable effect on the systems involved here.) On the other hand, in state (e) the temperature as well as the velocity cannot be uniform, that is to say, identical for all active bodies leaving the system.

The system transfers to the outside by radiation and conduction a certain quantity of heat (mQ_R) in unit time,² Q_R denoting the quantity of heat in question referred to unit mass of fuel consumed. The quantity of heat Q_R may be decomposed in two parts: a part Q_r representing the heat given off by the active bodies to the intermediary

²In the present report all energy or heat quantities are expressed by the same units, which eliminates the necessity for indicating the mechanical equivalent of heat in the formulas. The unit of work, or of heat, used here is, theoretically, the kilogram.

bodies with which they are related, and a part Q_f representing the supplementary heat transferred to the outside medium (ambient fluid) by the intermediary bodies and which essentially corresponds to the work of mechanical friction and to the passive resistance of the propulsion system (that is, resistance stemming from the intermediary bodies).

The internal energy will be denoted by mU and the volume of the active bodies consumed in unit time by mV .

If these active bodies went through the cycle in a fixed heat engine where their introduction and evacuation proceeded in the same physical and chemical state as in the propulsion system under consideration and without appreciable kinetic energy, and if this cycle were completed with the same transfer of heat (mQ_r) from active bodies to adjacent bodies, the indicated energy which these active bodies supplied in this engine per unit mass of consumed fuel would be

$$C_i = (U + pV)_a - (U + pV)_e - Q_r \quad (2)$$

This work or energy shall be called the indicated energy of the thermodynamic cycle of the active bodies in the system in question, per unit mass of fuel consumed. The corresponding effective energy C_{eff} will be the preceding term decreased by C_f , that is to say, minus the work of friction and passive resistance of the system, also referred to unit mass of fuel consumed. Hence, since $Q_r + Q_f = Q_R$,

$$C_{eff} = (U + pV)_a - (U + pV)_e - Q_R \quad (3)$$

It is by this work, which depends only on the physical and chemical state of admission (a) and evacuation (e) of the active bodies in the propulsion system and on the total exchange of heat between the system and the surrounding fluid, that the thermodynamic cycle of the active bodies in the system is characterized in the system under consideration.

By the same process, the thermal efficiency of the system shall be defined by the ratio

$$\boxed{\eta_{th} = \frac{C_{eff}}{L}} \quad (4)$$

This is an actual efficiency. An indicated thermal efficiency could be defined by the ratio

$$\eta_i = \frac{C_i}{L} \quad (5)$$

and a mechanical efficiency of the system by the ratio

$$\eta_m = \frac{C_{eff}}{C_i} = \frac{\eta_{th}}{\eta_i} \quad (6)$$

These definitions, obviously conventional, have the great advantage of being in keeping with those universally adapted in the study of ordinary heat engines.

Theoretically, only the thermal efficiency η_{th} defined by equation (4) is being considered here.

In addition, it may be recalled that the term C_{eff} defined by equation (3) may be put in more detailed form

$$C_{eff} = L - Q_c - Q_e - Q_R \quad (7)$$

where $L = (U + pV)_a - (U + pV)_f$ is the low heat value of the fuel (that is, of its unit mass), at the conditions p_a, T_a at the above-mentioned state (f) corresponding to these conditions and to a complete combustion, assuming zero condensation of water vapor.

$Q_c = (U + pV)_{f'} - (U + pV)_f$ is the heat loss due to incomplete combustion, the state (f') being that of the products of real combustion, reduced to p_a, T_a , assuming zero condensation of water vapor.

$Q_e = (U + pV)_e - (U + pV)_f$ is the heat loss due to the exhaust, that is, the sensible heat of the active bodies in the state of evacuation (e) with respect to the outside medium.

Equation (3) put in the form (7) is simply the classical heat balance of stationary heat engines in an atmosphere at (p_a, T_a) .

3. Definition of the propeller efficiency.

The thermal efficiency η_{th} that characterizes the thermodynamic utilization of the fuel in the propulsion system is defined by the same process. It is, in short, the engine which thus has been characterized in special fashion.

To characterize the propeller it is assigned an efficiency, η_p , termed propeller efficiency, defined by the condition that its product

by thermal efficiency η_{th} represents the over-all efficiency η_g defined by the relation (1):

$$\boxed{\eta_g = \eta_{th} \eta_p} \quad (8)$$

By definition the efficiency of the propeller is then:

$$\boxed{\eta_p = \frac{TV}{m\eta_{th}L}} \quad (9)$$

4. Application of these definitions to a classical case.

To demonstrate that the foregoing generalized definitions correspond, as desired, to the current notions used for ordinary propulsion systems, suppose that they are applied to an engine nacelle of an airplane or a dirigible equipped with an engine and a propeller.

In this case the power effectively transmitted to the propelled system is (TV), T denoting the effective thrust, that is, that which the nacelle transmits through its attachments to the propelled system (this thrust is equal to the actual propeller thrust measured over the hub, less the aerodynamic resistance of the nacelle when the propeller rotates).

If η_{th} is the effective thermal efficiency³ of the engine and m the fuel consumption in unit time, the effective horsepower transmitted by the engine to the propeller is ($m \eta_{th} L$) and the propeller efficiency η_h , in the usual sense, is the ratio of the effective power TV to this horsepower supplied by the engine

$$\eta_h = \frac{TV}{m\eta_{th}L}$$

It is readily seen that this ratio is identical with the propeller efficiency η_p defined by equation (9).

³In this term the kinetic energy of the air in the air intake of the carburetor and that in the burnt gases at exhaust pipe exit are ordinarily disregarded. These kinetic energies are, in fact, for practical purposes negligible, but in the general study of propulsion systems made here, exact allowance is necessary.

P A R T I

PROPULSION SYSTEMS WITH DIRECT AXIAL REACTION

ROCKETS AND ROCKETS WITH THRUST AUGMENTATION

The types of systems considered can be divided into two categories which will be studied in succession under the following headings:

A. Explosive rockets, fed by means of a fuel containing its own combustion air.

B. Conventional-fuel rockets, that is, deriving its carburation air from the outside atmosphere.

Lastly, under C, the principal results obtained are summarized and the rockets compared to the engine-propeller system.

A. EXPLOSIVE ROCKETS

Chapter I - True Explosive Rocket

5. Definition.

This type of rocket corresponds to the diagram of figure 1. It comprises a shell of streamlined shape, truncated at the rear, along the exhaust section S_e on which the cavity or core of the rocket opens. This core carries a combustion chamber containing the stored explosive and an expansion nozzle connecting the chamber with the discharge orifice.

Ignition is by means of some kind of detonator or primer, actuated, for example, electrically.

The gases exhaust toward the rear with a certain relative speed w_e .

Assume a steady state, an axial and uniform translation of rocket at speed V , and, lastly, uniform pressure and speed in the exhaust section S_e .

The rocket and the bodies contained in it are, on the outside, subjected

(a) To the resultant R_e of the force of the surrounding fluid on the shell, obviously along the axis and counted positive in the direction opposite to V , that is, in the direction of the resistances

(b) To the outside pressure p_e acting on the exhaust section S_e

(c) Lastly, to the connecting force with the propelled system, equal and opposite to the thrust T of the rocket.

6. Calculation of thrust and over-all efficiency.

The momentum theorem projected along the direction of speed V is now applied to the rocket and to the bodies contained in it at instant t , in absolute motion and during time interval dt .

With m as the mass of explosive consumed per unit time,

$$(-R_e + p_e S_e - T)dt = -mV dt + m(V - w)dt$$

hence

$$T = p_e S_e - R_e + mw_e \quad (10)$$

Consider first the term $(p_e S_e - R_e)$. The term $(-R_e)$ is the resultant of all superficial forces of the surrounding fluid projected along speed V , that is, of the outside air on the outside shell of the rocket.

These forces are decomposed, at each point, into a normal and into a tangential force.

At low speed it may be admitted, on one hand, according to the boundary layer concept, that the normal force is practically equal to the local pressure of a perfect fluid of the same rate of flow provided that the body is adequately streamlined and, on the other, that the tangential force arises from the contact friction of the fluid layer which adheres to the body. In these conditions, it is easily seen that the resultant along V of the normal forces at the shell, augmented by $(p_e S_e)$, represents the resultant along V of the normal forces at the shell, less the unit pressure p_e at each point. In other words, it is the geometric sum projected on V of the high and low pressures (with respect to p_e) acting on the outside surface of the shell. Referring to pressure measurements on the surface of streamlined bodies tested at fairly small Reynolds numbers, it is apparent that the resultant in question is very small compared to the thrust T when the latter has an appreciable value. The term $(p_e S_e - R_e)$ is therefore reduced when T is considerable at the resultant along V of the forces

of friction of the surrounding fluid on the shell. This resultant is manifestly negative and its value is, therefore, relatively low, when T is sufficiently high.

At high speed the viscosity of the air gives precedence to its compressibility and the tangential forces are practically negligible. But nothing is known then of the pressure distribution over the envelope of the rocket and the question must be left to experiment. However, it would seem possible to establish a certain analogy between the rocket and an artillery shell, at least for head resistance, that is to say, on the portion located forward of the section AB where the cylindrical part of the body begins (fig. 2).

The difference between the rocket and the shell is indicated especially by the flow of gas ejected by the former, which tends to regulate the air flow so profoundly disturbed behind the base of the shell.

Lack of knowledge on the aerodynamic resistance of the several parts of a shell does not permit any conclusions to be drawn from this comparison other than the probability of smaller magnitude of the term $(p_e S_e - R_e)$ corresponding to the rocket with respect to the resistance of a projectile of suitable form.

Besides, the axial propulsion rockets considered can have no very high speed and the remarks about the rockets with very high speed are described in the second part of the present report where jet propellers, fitted with rockets at the tips of the blades, are discussed.

The term $(p_e S_e - R_e)$ which, according to the foregoing, is, in general, supposed to be negative, that is to say, to represent an effective resistance to advance, can be put in the form

$$p_e S_e - R_e = -c_{re} \frac{\rho_a}{2} S_e V^2 \quad (11)$$

c_{re} signifying the aerodynamic coefficient of this resistance referred to the density ρ_a of the surrounding air, to the exhaust section S_e , and the square of the speed V .

On the other hand, with ρ_e denoting the density of the exhaust gases the mass discharge of the rocket has the value

$$m = \rho_e S_e w_e \quad (12)$$

The thrust T evaluated by formula (10) is, in consequence, put in the form

$$T = \frac{1}{2} \rho_a S_e V^2 \left[2 \frac{\rho_e}{\rho_a} \left(\frac{w_e}{V} \right)^2 - c_{re} \right] \quad (13)$$

This formula enables the order of magnitude of the two terms in parenthesis to be known.

Supposing that the coefficient c_{re} is of the order of magnitude of the coefficients of frontal resistance of a suitably streamlined body at forward speeds of the order of 100 to 1000 km/h. In these conditions, c_{re} probably ranges between 0.05 and 0.12, depending on the shape and speed.

As to the exhaust gases, for explosives such as smokeless powder, their density is near that of the air (in reality, less than 5 percent) at equal temperature. If the absolute exhaust temperature is two or three times that of the surrounding medium, the ratio ρ_e/ρ_a is then of the order of 1/3. But the ratio w_e/V , for V ranging between 100 and 1000 km/h or between 28 and 280 m/sec is at least equal to 3 and may easily reach a value of 30 to 50, if the speed V is low enough. Hence, the first parenthesized term of (13) is, at least, of the order of 6, while the second term would be of the order of 0.05 to 0.12.

Therefore, it is almost certain that, for the applications in view, the term in c_{re} can be neglected without appreciable error and the expression for the thrust T reduced to

$$T = \rho_e S_e w_e^2 = m w_e \quad (14)$$

This expression is adopted here, to simplify matters.

The propulsive efficiency defined by (9) then takes the simple form

$$\eta_p = \frac{V^2}{\eta_{th} L} \frac{w_e}{V} \quad (15)$$

The over-all efficiency η_g of the rocket considered as system of propulsion is by virtue of the qualifying equation (8),

$$\eta_g = \frac{V^2}{L} \frac{w_e}{V} \quad (16)$$

The above expressions of T , η_p , and η_g introduce the relative speed of exhaust w_e , which is determined next.

7. Calculation of the relative speed of the exhaust.

Consider the system formed by the rocket and the bodies contained in it (explosives and combustion gases) at instant t upon assuming a steady state at the same time for the translation of the rocket, the combustion of the powder, and the exhaust of the gases from the rocket.

This state is steady neither for the distribution of the bodies in the system, since the space occupied by the explosive decreases progressively, nor for the mass of bodies contained in the rocket.

Let us apply the principle of the conservation of energy to the system in question during time interval dt and with respect to the axes fixed in the rocket and consequently, actuated by an absolutely uniform forward speed. The energy of the outside forces is reduced to $(-p_e S_e w_e dt)$. The variation of the internal energy of the system is

$$m dt(\underline{U}_e - \underline{U}_0)$$

where \underline{U} = internal energy per unit mass of active bodies (explosive), \underline{U}_e refers to the assumedly uniform state of exhaust gases and \underline{U}_0 refers to the state of the explosive before combustion, a state assumed to be unaffected by the combustion in adjacent sections.

The change in kinetic energy relative to the system is reduced to $m dt \frac{w_e^2}{2}$.

The system necessarily transmits a certain amount of heat to the outside through the envelope of the rocket, but, for simplification, this exchange of heat can be disregarded; the rocket can be practically likened to a rigorously adiabatic system.

In these conditions the principle of conservation of energy is

$$-p_e S_e w_e = m \left[\underline{U}_e - \underline{U}_0 + \frac{w_e^2}{2} \right]$$

or with \underline{V}_e denoting the specific volume of the exhaust gases

$$[m \underline{V}_e = S_e w_e]$$

and, by dividing by m

$$\frac{w_e^2}{2} = (\underline{U} + p\underline{V})_e - \underline{U}_0 \quad (17)$$

The initial state of the explosive before combustion can be likened to its state in the conditions p_a, T_a of the surrounding medium. The explosive is then a solid whose unit mass possesses the internal energy \underline{U}_a and the volume \underline{V}_a .

The product ($p_a \underline{V}_a$) being entirely negligible with respect to ($p_e \underline{V}_e$), the \underline{U}_0 in equation (17) is replaced by $(\underline{U}_a + p_a \underline{V}_a)$, so that

$$\frac{w_e^2}{2} = (\underline{U} + p\underline{V})_e - (\underline{U} + p\underline{V})_a \quad (18)$$

The kinetic energy of the gases at exhaust from the adiabatic rocket is thus found to be equal to that which previously was called the effective energy of the thermodynamic cycle.

In consequence of which, according to the equation (4) which defines the thermal efficiency η_{th} , we can put

$$w_e^2 = 2\eta_{th}L \quad (19)$$

Utilizing the thus-obtained value of w_e , equations (14), (15), and (16) can be written in the form

$$T = 2\rho_e S_e \eta_{th} L = m \sqrt{2\eta_{th}L} \quad (20)$$

$$\eta_p = V \sqrt{\frac{2}{\eta_{th}L}} \quad (21)$$

$$\eta_g = V \sqrt{\frac{2\eta_{th}}{L}} \quad (22)$$

8. Over-all efficiency.

Consider formula (22) in which, as will be recalled, the effect of coefficient of aerodynamic resistance c_{re} of the rocket, which constitutes an acceptable approximation as long as V is not too high, has been neglected.

The expression of the over-all efficiency thus established indicates that the latter is dependent only on:

Speed V

Thermal efficiency η_{th}

The heat value L of the explosive

It further shows that η_g increases in proportion to the speed V and the square root of the thermal efficiency. It lastly shows that subject to the reservation indicated above concerning the approximation effected by disregarding the coefficient c_{re} , the over-all efficiency can increase indefinitely at the same time as V .

This seemingly paradoxical conclusion is justified when noting that the rocket consumes, during flight, a part of its mass corresponding to the explosive transformed in exhaust gas.

Thus, in order to produce the absolute energy of propulsion TV , it consumes not only the fraction η_g of the available energy mL of the consumed explosive, but also the absolute kinetic energy $m \frac{V^2}{2}$ of the mass m of the explosive at the moment of its utilization.

If the effective power TV is referred to the sum $m \left(L + \frac{V^2}{2} \right)$ rather than to mL , an energy efficiency η_e can be defined, whose value would be

$$\eta_e = \frac{TV}{m \left(L + \frac{V^2}{2} \right)} = \frac{2V\sqrt{2\eta_{th}L}}{2L + V^2}$$

of which the maximum, in function of V (that is, for η_{th} and L constant) occurs at $V = \sqrt{2L}$ and has the value

$$\eta_e = \sqrt{\eta_{th}}$$

This ratio has then the same limit as $\sqrt{\eta_{th}}$ and remains less than unity when conceding that the thermal efficiency η_{th} itself cannot exceed unity.

This remark points out the importance of the admonition voiced at the beginning of the report (cf. article 1) on the subject of

conventional and not perfectly rational character of the definitions adopted for the several efficiencies and conformable to more extensive usage.

On examination of the values attainable for the over-all efficiency η_g of the rocket, evaluated by formula (22), it is found that the heat value of explosives depends upon their nature:

For black powder, L is of the order of 650 cal/kg (or in the chosen units, 2,760,000 kgm per unit mass).

For powders such as smokeless powder, powder B, colloidal powder, or gun-cotton, L is of the order of 1050 to 1250 cal/kg.

For the calculations, a powder B with a heat value L equal to 1200 cal/kg or 5,000,000 kgm/unit mass is considered.

The thermal efficiency η_{th} of the rocket depends upon the pressure of combustion in the chamber, say, for example, 0.20, 0.40, and 0.60.

In these conditions the over-all efficiency η_g of the rocket reaches the values given in table I, at different speeds V .

TABLE I
OVER-ALL EFFICIENCY OF ROCKET
(Powder B; $L = 5,000,000$ kgm)

$\eta_{th} =$ $V =$	0.20	0.40	0.60
40 m/sec (144 km/h)	$\eta_g = 0.011$	0.016	0.019
80 m/sec (288 km/h)	0.022	0.032	0.039
120 m/sec (432 km/h)	0.033	0.048	0.058
160 m/sec (576 km/h)	0.044	0.064	0.078
200 m/sec (720 km/h)	0.055	0.080	0.098

For black powder ($L = 2,760,000$), the figures of the table must be multiplied by 1.30.

Hence, it is seen that, even with favorable thermal efficiency (η_{th} ranging between 0.40 and 0.60) and up to speeds of the order of 700 km/h, the over-all efficiency of the rocket is less than 10 percent (13.5 percent for black powder) while the engine-propeller system used in airplanes reaches a thermal efficiency (engine) of from 25 to 30 percent and a propulsive efficiency (propeller) of 60 to 75 percent, that is, an over-all efficiency ranging between 15 and 22.5 percent.

Thus the powder rocket is characterized, a priori, by mediocre or insufficient over-all efficiency, which can be improved only by raising the thermal efficiency or by restricting the use of the rocket to the range of very high speed (above 700 km/h).

9. Organic structure of the perfect rocket - thermal efficiency - development of simple formulas for the perfectly adiabatic rocket operating in a uniform state.⁴

The combustion being assumed adiabatic and realized at constant pressure p_c starting from temperature T_a , the combustion temperature T_c is defined by

$$L = \int_{T_a}^{T_c} C dT \quad (23)$$

C = the specific heat at constant pressure of the products of combustion, assuming no water vapor being condensed.

The adiabatic expansion of gas from p_c to p_e , assumedly effected according to the reversible adiabatic (or isentropic) process, leads to the final temperature T_e such that

$$T_e = T_c \left(\frac{p_e}{p_c} \right)^{\frac{\gamma-1}{\gamma}} \quad (24)$$

γ denoting the ratio of specific heat C/c of the products of combustion.

⁴Incidentally, it seems timely to recall that the question of flow of gases of powder in nozzles had occupied a number of French scientists during the period from 1914 to 1918 in connection with projectiles called rocket projectiles as well as with recoil cylinders for cannons (muzzle brakes).

On the other hand, the effective energy of the thermodynamic cycle is

$$\eta_{th} L = \int_{T_e}^{T_c} C dT \quad (25)$$

In comparing (24) and (25) and noting that T_a and T_e are largely of the same order with respect to T_c which is very great, that is to say, that the mean value of C between T_a and T_c is comparable to its average value between T_e and T_c , we can put

$$\eta_{th} = \frac{T_c - T_e}{T_c - T_a}$$

or, with allowance for (24) and putting λ = ratio of expansion in the nozzle = p_c/p_e

$$\eta_{th} = \frac{T_c}{T_c - T_a} \left[1 - \frac{1}{\lambda^{\frac{\gamma-1}{\gamma}}} \right] \quad (26)$$

This relation shows that the thermal efficiency of the perfect adiabatic rocket (whose combustion temperature T_c is according to (23) independent of the pressure when the initial temperature T_a is given) depends only upon the ratio of expansion λ in the nozzle of the rocket.

The speed of exhaust from the nozzle exit is given by (19). Next, the thrust referred to unit surface of the discharge section of the nozzle is calculated. This intensity of thrust $t = \frac{T}{S_e}$ is according to (20)

$$t = 2\rho_e \eta_{th} L$$

The exhaust density follows from the equation of state of the powder gases

$$p_e \left(\frac{1}{\rho_e} - \alpha \right) = RT_e$$

α being the covolume of the powder gases and R a constant.

Calculation readily yields the formula

$$t = \frac{T}{S_e} = \frac{2\eta_{th}L}{\alpha + \frac{RT_c}{p_e \lambda^{\frac{\gamma-1}{\gamma}}}} \quad (27)$$

For values of λ below 500 (that is, for $p_c < 500 \text{ kg/cm}^2$ if $p_a = 1 \text{ kg/cm}^2$), the covolume α can be disregarded in the preceding relation which then is reduced, after simplifications, to

$$t = 2 \frac{p_e L}{R(T_c - T_a)} \left[\lambda^{\frac{\gamma-1}{\gamma}} - 1 \right]$$

These formulas are applied to the case of a rocket charged with powder B.

The characteristics of this explosive are according to the information furnished by various authors

$$L = 5,000,000$$

$$R = 309$$

$$\gamma = 1.25$$

$$T_c = 2,450^\circ \text{ (on the premise of } T_a = 273 + 15 = 288^\circ \text{)}$$

The data of the thermal efficiency η_{th} , speed w_e , exhaust temperature T_e , and, lastly, the ratio t/p_e of the intensity of thrust t to the exhaust pressure for different expansion ratios $\lambda = p_c/p_e$ are reproduced in table II.

TABLE II

λ	η_{th}	w_e	T_e	t/p_e
		m/sec	degrees	
10	0.415	2,035	1,545	8.95
50	.615	2,480	1,120	16.7
100	.680	2,605	975	23.3
200	.740	2,720	850	29.0
300	.770	2,770	785	32.9
400	.790	2,805	740	35.6
500	.805	2,835	705	38.0

The data in the table are only approximate, especially for the low values of λ because the calculation was simplified by replacing the specific heat C by an average value and assumed the same in (23) and (25). This is evidently inaccurate when T_e differs very much from T_a , that is, according to the figures in the preceding table, at least as much as the difference between 100 and λ .

But this approximation which plays no part in the calculation of T_e is sufficient to show that values of the order of 0.60 in thermal efficiency can be reached only at a combustion pressure of from 50 to 100 times the exhaust pressure.

The values of t/p_e are interesting to the extent of showing that this fictitious perfect adiabatic rocket makes it possible to obtain a very substantial thrust with a relatively small exhaust section. Thus for a rocket with an internal pressure of 100 kg/cm² exhausting into standard atmosphere ($p_e = 1$ kg/cm²) a section of 43 cm² (or a circular exhaust orifice of 7.4 cm in diameter) is sufficient to secure a thrust of 1 ton.

However, this rocket is, by assumption, an ideal engine, the realization of which raises certain difficulties and involves certain efficiency losses which are to be examined next.

10. Study of real rocket - special difficulties - obtainable efficiency

(1) Existence and stability of state.- In the foregoing, the existence of a so-called steady state of rocket operation had been assumed. This state is now determined.

Suppose that s is the surface of combustion of the explosive. According to the classical works of interior ballistics, the volume in mass m_1 of combustion gases can be written in the form

$$m_1 = k s p_c^{0.7} \quad (29)$$

k denoting a characteristics coefficient of the explosive and p_c the pressure in the combustion chamber.

The exponent 0.7 is a matter of dispute; certain authors give it a higher value, rounded to unity. On the other hand, the known tests refer especially to very high pressures.

The volume m_1 must be equal to that of the gases evacuated by the nozzle and it is supposed that the latter is of the converging-diverging type, as is necessary for the applications in view where p_c

is always twice as high as the external pressure. In the throat of the nozzle, the flow is determined solely by pressure p_c and is found to be independent of the pressure at the outlet ($p_e < p_c/2$).

Disregarding the covolume α of the gases of the explosive, which is legitimate for the pressures $p_c < 500 \text{ kg/cm}^2$ in question, it is readily calculated that the pressure in the throat is $p_0 = 0.555 p_c$ and the corresponding temperature $T_0 = 2,175^\circ$ for powder B ($\gamma = 1.25$). The volume in mass m_2 through the throat of section S_0 is given (in M.K.S. units) by the formula

$$m_2 = \rho_0 S_0 w_0 = \rho_0 S_0 \sqrt{\frac{\gamma p_0}{\rho_0}}$$

or

$$m_2 = 0.000756 S_0 p_c \quad (30)$$

Equating (29) and (30) gives

$$k_{sp_c}^{0.7} = 0.000756 S_0 p_c \quad (31)$$

which determines the pressure p_c of the state.

This state is stable as proved by the following argument. Plotting the volume m_1 of combustion of the powder obtained by (29) and the volume m_2 of the nozzle obtained by (30) against p_c (fig. 4), these curves intersect at a point M.

This point M represents the state which necessarily exists and which according to the shape of the curves is unique, so far as the assumptions which had to be made in the establishing of formulas (29) and (30) are verified when the pressure p_c changes from zero to its value of the corresponding state in point M. The state is stable because, when the pressure rises, the volume m_2 of the nozzle exceeds that of the combustion of the powder and the pressure tends to decrease.

The inverse or reciprocal effect is produced when the pressure decreases. The thus demonstrated stability of the state of functioning is in good agreement with the statements made about the operation of rockets commonly utilized in fireworks. However, this stability supposes a law of the form (21) verified for the rate of combustion of the powder, with an exponent of p_c less than unity. Now, such an

assumption is not admitted without dispute⁵ and there is a serious reason for making some reservations on the stability, in general, of the functioning of explosive rockets.

(2) Hypothesis on the initial state of the explosive in the rocket.- The previously developed formulas suppose that the powder, prior to its combustion, remains in the core of the rocket at the original temperature T_a . It is largely by reason of this assumption that the combustion temperature T_c is independent of the combustion pressure p_c as represented by (23). In fact, the rocket being heated by conductivity, it is possible that the temperature of the still-unburnt powder increases progressively. This fact gives rise to a risk of spontaneous and accelerated ignition falsifying the normal law of the rate of combustion. So in this respect there might be some fear of certain risks of inopportune explosion of the rocket or instability of its normal state of operation. Incidentally, there come to mind certain unexpected explosions of rockets in the course of automobile rocket tests and also in rocket-equipped glider tests made in a foreign country (Germany).

(3) Losses through the walls and in the nozzle - practical efficiency of the rocket.- The previously computed thermal efficiencies refer to an assumedly adiabatic rocket. But the corresponding temperature $2,450^\circ$ for powder B is excessive and inadmissible for a continuous and prolonged state of operation. Necessarily, it must tolerate a certain loss through the walls in order to limit the temperature in the combustion chamber to a value compatible with the resistance of the latter, under the pressure to which it is assumed to be subjected.

By way of example, suppose that the absolute pressure and temperature in the combustion chamber of the rocket with power B is limited to 100 kg/cm^2 and 1800° . The exhaust temperature T_e drops then from 975° to 718° and the thermal efficiency, perfect expansion being assumed, drops from 0.680 to 0.505. (The heat loss due to exhaust is reduced from 32 percent to 20 percent, but this is supplemented by a 29.5 percent loss through the walls.)

Admitting, very optimistically, that the efficiency of the expansion nozzle is 95 percent, the effective thermal efficiency of the rocket is then 0.95×0.505 , or 48 percent.

On a reasonable estimate, the thermal efficiency of an explosive rocket cannot be over 45 to 50 percent.

⁵It is pointed out that, according to certain experiments, the law (21) should be replaced by a linear law in the low-pressure range.

In these conditions, it is concluded (cf. table I) that up to speeds of the order of 700 km/h the simple rocket with explosive is nearly two times less advantageous than the normal engine-propeller system.

Ascribing to the latter an over-all efficiency of $0.28 \times 0.75 = 0.21$ which is representative of a good engine and a well adapted propeller, the equality between it and the explosive rocket is obtained at a speed determined by (22), and which is

$$V = 0.21 \sqrt{\frac{L}{2\eta_{th}}} \quad (32)$$

For powder B ($L = 5,000,000$), with $\eta_{th} = 0.50$, the equivalent speed is

$$V = 470 \text{ m/sec} = 1690 \text{ km/h}$$

For black powder ($L = 2,760,000$), with $\eta_{th} = 0.50$, the equivalent speed is

$$V = 350 \text{ m/sec} = 1,260 \text{ km/h}$$

At such speeds, the propeller is no longer a satisfactory propelling medium and this practically leads to values below the preceding values of the speeds where the explosive rocket may offer the same over-all efficiency as the propeller-engine system.

(4) Effect of altitude on the operation of the rocket.- Equation (31) shows, when assuming that the surface s of the combustion is constant, that the outside conditions do not effect the pressure of combustion p_c and that, in consequence, the latter remains constant. The temperature of combustion can, moreover, be assumed constant by admitting that the cooling of the rocket can be suitably controlled. As a result the volume of the rocket, determined by the conditions upstream and in the throat of the nozzle which remain constant, is independent of the outside conditions.

But the speed of the jet at the exit of the expansion nozzle is, theoretically, affected by the external conditions.

Two cases are distinguished:

(a) The expansion nozzle has a fixed, divergent opening and is of constant length. In this event, the nozzle, adapted to operate at a certain altitude is either too long or too short for a higher or lower altitude, or in other words, for a higher or lower outside pressure.

The flow in the divergent section becomes imperfect, and, as a rule, the efficiency of the rocket must decrease.

(b) The expansion nozzle has a controllable divergent opening, so that, when the altitude increases, that is, as the outside pressure decreases, the expansion ratio increases. The volume of the rocket being constant, the thrust T increases as w_e , that is, as $\sqrt{\eta_{th}}$ increases.

Referring to equation (26) which gives an approximate expression for η_{th} , it is seen that this term increases with λ , hence as the outside pressure decreases. The thrust increases thus with the altitude, which enables a rocket-propelled airplane to fly at a state of smaller lift and, consequently, to increase the speed faster than the proportionality to $1/\sqrt{\delta}$ (δ being the corresponding atmospheric density at the particular altitude).

Moreover, by virtue of (22) the over-all efficiency η_g of the rocket increases with the altitude, since η_{th} and V increase in the same conditions.

But it should be remembered that these results, while obviously quite interesting, assume the rocket nozzle controllable in flight, and this raises certain difficulties of realization.

11. Utilization of rocket for airplane propulsion - control of the rocket

The thrust of a rocket, at constant altitude, is independent of the forward speed V when the surface of combustion of the powder is constant and the nozzle is of constant shape.

The problem then is to vary the thrust either at will of the pilot or in relation to circumstances in flight.

The means that can be envisaged to this effect for an explosive rocket are the following:

(a) Variation of the surface of combustion of the powder, a procedure which can be applicable for a defined law of advance of variation, but which seems inapplicable for a law that is arbitrary or changeable at will by the pilot

(b) Modification of the cooling of the rocket, to reduce the speed of exhaust. This effect, obtained to the detriment of the thermal efficiency, is disadvantageous.

(c) Modification of the shape of the nozzle, that is of the throat section. This procedure, difficult to realize for a nozzle of revolution will, strictly speaking, be acceptable for nozzles of

rectangular section. The effect of varying the throat section is a modification of the combustion pressure and, in consequence, of the thermal efficiency. This is also, in certain conditions, a disadvantageous procedure.

(d) Variation of the number of rockets in operation. While it is a comparatively easy matter to start new rockets by means of electric ignition switches, it is much more difficult to visualize the extinction, at will, of a rocket in operation. This procedure, theoretically perfect as to the nonexistence of an effect on the thermal efficiency, runs against serious practical difficulties as regards its reversibility.

Finally, only procedures (b) and (c) hold some promise of practical solution of controlling the thrust supplied by an explosive rocket, but to the detriment of the thermal efficiency in certain cases.

12. Range of airplane propelled by powder rocket - Comparison with orthodox airplane (engine and propeller)

Suppose that the thrust of the rocket is by some means controllable without changing the efficiency.

If the aircraft flies at a constant incidence for which its lift-drag ratio is c_x/c_z , its drag for a total weight P is Pc_x/c_z and it is this drag which must balance the thrust of the rocket, given by formula (20). Therefore

$$T = m \sqrt{2\eta_{th}L} = Pc_x/c_z \quad (33)$$

The weight P at instant t is equal to the total weight at take-off P_0 less the weight C of the explosive already consumed. During time interval dt , the airplane covers in still air the elementary distance (in km)

$$dx = \frac{V dt}{1,000}$$

and consumes a weight of explosive equal to

$$dC = mg dt = m \frac{9,810}{V} dx \quad (34)$$

By elimination of m between (34) and (33) and allowance for $P = P_0 - C$

$$dx = \frac{dC}{P_0 - C} \frac{c_z}{c_x} \frac{V \sqrt{2\eta_{th}L}}{9,810} = \frac{9,810}{L} \frac{c_z \eta g}{c_x} \frac{dC}{P_0 - C}$$

The range X (in km) in still air is obtained by integrating the preceding relation between 0 and C_0 , with C_0 representing the total weight of fuel carried at take-off.

Hence

$$X = \frac{L}{4,270} \frac{c_z \eta_g}{c_x} \log \frac{P_0}{P_0 - C_0} \quad (35)$$

There is an identical formula for an airplane with engine and propeller in which L represents the heat value of the fuel and η_g the over-all efficiency of the engine-propeller unit. Assume now that the engine-propeller unit is removed from this airplane and a powder rocket installed, without affecting the fineness ratio of the airplane nor its capacity in fuel load.

The original range X' becomes obviously X so that

$$X = X' \frac{L \eta_g}{L' \eta_g'} \quad (36)$$

The over-all efficiency of a good engine-propeller combination is $0.28 \times 0.75 = 0.21$. The heat value of the fuel is around 11,000 cal/kg.

For a rocket, an η_g value of the order of 0.03 to 0.04 may be counted on at ordinary speeds (200 to 300 km/h), and of the order of 0.09 to 0.11 at very high speeds (700 to 800 km/h). On the other hand, the heat value L of the explosive is only 1/17 or 1/9 of that of fuel, depending upon whether the explosive is black powder or powder B. Thus, even at higher speed actually reached in current practice (300 km/h), the powder rocket reduces the range 1 or 2 percent of its original value, depending upon whether ordinary black powder or powder B is used.

13. Conclusions

The use of the pure explosive rocket as normal means of propulsion is predicated upon the solution of two additional but important problems:

- (1) Stability of state of operation
- (2) Possibility of easy and exact control

Assuming these two conditions to be satisfied, the study of the over-all efficiency shows that it cannot attain a value comparable to

that of an engine-propeller system except at very high speeds, of the order of 1,000 km/h. At speeds of the order of 300 km/h, the over-all efficiency does not exceed $1/4$ of that of the engine-propeller system.

Lastly, the range of a rocket airplane at ordinary speeds is insignificant by reason of the low over-all efficiency of the rocket and the heat value of the explosives.

These two constitutional defects absolutely prohibit any application of the explosive rocket for normal airplane propulsion. At best, it may be reserved, outside of interstellar navigation where it constitutes the only propeller available, for the propulsion of special devices at very high speed more nearly to projectile than an airplane. In this field, which is of interest to the artillery, some interesting and fruitful studies are offered.

14. General remarks on the efficiency of direct reaction propulsion systems

From the above calculations and examples it is clear that the true powder rocket constitutes a propulsion system of very mediocre efficiency, at least as long as V does not assume very high values. This is largely due to the fact that the speed of exhaust w_e is extremely high with respect to the forward speed V .

This is a characteristic common to all systems of propulsion by direct reaction based upon the ejection of fluid masses.

The propulsion systems to be considered (with direct or longitudinal reaction) always operate by communicating to one or several masses of fluid, ejected in continuous or periodical manner, a corresponding speed opposite to the forward speed V . The liquids in question are of two kinds: they either arise from the active bodies provided for on the rocket and converted by combustion (such as powder gases in an explosive rocket, for example) or come from the infinite surrounding medium from which they are taken.

Let $m_1, m_2 \dots$ be the volume of the liquids of the first category; $w_1, w_2 \dots$ their corresponding speed of exhaust directed downstream; and $m_1', m_2' \dots, w_1', w_2' \dots$ the corresponding quantities for the liquids of the secondary category, while assuming that with respect to the exhaust orifices the surrounding pressure is comparable to the general pressure p_a of the outside medium. In these conditions the momentum theorem indicates that the thrust T developed by the system (or more accurately the real thrust T diminished by the aerodynamic resistance of the propulsion system in

the surrounding medium, which, in general, is altogether negligible) has the value

$$T = \sum m_i w_i + m_j' (w_j' - V) \quad (37)$$

On the other hand, according to the law of conservation of energy and by definition of the thermal efficiency η_{th}

$$\eta_{th} \sum m_i L = \frac{1}{2} \sum [m_i w_i^2 + m_j' (w_j'^2 - V^2)] \quad (38)$$

This relation states that the actually utilized fraction of the heat value of fuel consumed in unit time has been transformed by variation of the corresponding kinetic energy of the active bodies.

To characterize the various efficiencies, the definitions of the over-all efficiency η_g and of the propeller efficiency η_p fixed at the beginning of this report and expressed by (1) and (8) are slightly modified.

In place of the over-all efficiency η_g consider the energy efficiency η_e defined by the ratio of effective power TV to power supplied to the system, which is composed, on the one hand, of the heat value $\sum m_i L$ and, on the other, of the absolute kinetic power $\frac{1}{2} \sum m_i V^2$ of the consumed fuel

$$\eta_e = \frac{TV}{\sum m_i \left(L + \frac{V^2}{2} \right)} \quad (39)$$

Instead of propeller efficiency $\eta_p = \eta_g / \eta_{th}$, consider the analogous term obtained by replacing η_g by η_e . To avoid any confusion, the reaction efficiency η_r is expressed by

$$\eta_r = \frac{\eta_e}{\eta_{th}} = \frac{1}{\eta_{th}} \frac{TV}{\sum m_i \left(L + \frac{V^2}{2} \right)} \quad (40)$$

The thus defined efficiencies are more rational than those (η_g, η_p) used up to now. They differ only by the addition of the term $\frac{V^2}{2}$, a term which in general and for the ordinary applications, is

negligible with respect to L , which justifies the current use of η_g and η_p . The effect of adding the term $\frac{V^2}{2}$ is to make the indefinite increase of efficiency with V , to which the method of defining η_g or η_p leads, disappear.

Formula (40) with due regards to (38) assumes the form

$$\eta_r = \frac{TV}{\Sigma [m_i w_i^2 + m_j' (w_j'^2 - V^2)]} \frac{2\Sigma m_i L}{\Sigma m_i \left(L + \frac{V^2}{2} \right)} \quad (41)$$

Formulas (37), (38), (39), and (41) make for a convenient and fairly general discussion of jet propulsion as illustrated by several examples.

(1) Case of true explosive rocket. - There is no volume m_j' and the volumes m_i are reduced to one, that is m . When T and V are given, it is readily seen from the preceding formulas that

(a) m varies in the inverse sense of w

(b) To increase η_e , m must be reduced, hence w and η_{th} increased. The jet efficiency η_r remains then constant.

When the thermal efficiency η_{th} is given, η_e varies as η_r . The thrust T for given V increases with m , and η_e , as well as η_r , remains then constant.

If η_{th} and T are given, η_e varies necessarily as η_r and these two efficiencies, with m and w constant, depend only on the speed V . Both are maximum for $V = \sqrt{2L}$. If V increases indefinitely, both tend toward zero, while the efficiencies η_g and η_p defined by (1) and (8), increase indefinitely and can be made to increase to the interest of unlimited V .

By way of example the efficiencies η_e and η_r are plotted against V in figure 5 as solid curves.⁶ For a rocket with black powder or powder B, on the basis of a thermal efficiency η_{th}

⁶Concerning figure 5, it is noted that η_e remains considerably below unity (its maximum is equal to 0.707) but also that η_r exceeds unity, because $\eta_r = 2\eta_e$ for $\eta_{th} = 0.5$. This stems from the fact that η_r is a conventional efficiency. Only η_e has from the energy point of view, the sense of an "efficiency."

of 50 percent. The broken curves represent the corresponding efficiencies η_g and η_p which are practically equivalent to η_e and η_r up to very high speeds of the order of 1500 km/h.

In short, in the case of the powder rocket, the mediocrity of η_e and η_p arises from the inferiority of speed V with respect to the speed of exhaust, but, contrary to what there is a tendency at times to believe, no improvement can be obtained for a given thrust T and speed V by an increase in m and by a decrease, correlatively, of the speed of exhaust w . On the contrary, in these conditions an improvement in efficiency calls for a decrease in m and an increase in w . Or else, recourse may be had to the principle of the thrust augmentser tube, which is discussed in the following.

(2) Explosive rocket with augmentser tube. - For the sake of simplification suppose that there is only a powder volume m and one outside fluid volume m' , both moving at uniform and identical exhaust speed w .

Thrust T and speed V are given.

Then, in order to increase η_e , m must be reduced without increase in w according to (37).

Assume that the thermal efficiency η_{th} is constant and known. Equations (37) and (38) establish two relations between the three variables m , m' , and w . Consider m' and w as functions of m . It readily yields

$$\frac{dm'}{dm} = \frac{m'V^2 - (2m + m')w^2}{m(w - V)^2}$$

$$\frac{dw}{dm} = \frac{(m + m')w^2 + mVw - m'V^2}{m(m + m')(w - V)}$$

For positive thrust T , that is, for the system to be effectively propulsive it is necessary that

$$(m + m')w > m'V$$

The discussion of the change in η_e and η_r with m , m' , and w is summarized in the adjoining table III.

TABLE III

Value of w	Variation of η_e and η_r with m , m' , and w
	η_e and η_r increase (T and η_{th} being constants) when m decreases, that is, when:
$\frac{m'}{m + m'} V < w < V \sqrt{\frac{m'}{2m + m'}}$	m' decreases, hence, when w increases
$V \sqrt{\frac{m'}{2m + m'}} < w < V$	m' increases, hence w increases
$V < w$	m' increases, hence w decreases

The speed of exhaust w , being, generally, greater than V , it is seen that, as a rule, to increase η_e and η_r it is necessary to increase the volume m' of the fluid taken from the outside and to slow up the ejected jet. But, when w is lower than V , w being quite small or V very great, the preceding table shows that these conclusions are profoundly modified.

This point is reverted to again later on.

(3) Case of rocket with liquid fuel.- In this case the combustion air taken from the outside takes the place of the surrounding fluid passing through the tube of the explosive rocket with thrust augmentation. The discussion of the preceding case is entirely valid, with the exception that the heat value L involved is much more considerable.

(4) Case of the liquid-fuel rocket with thrust augmentation.- This case is analogous to the preceding one. The volume m' of fluid taken from the outside must then include, apart from the combustion air, the surrounding fluid on which the augments tube acts.

NOTE: It is for the obvious purpose of simplification that the number of categories of volume and speeds m_i , m_j' , w_i , and w_j' were reduced to a minimum. The formulas (37) to (41) lend themselves to the discussion of much more complicated cases.

CHAPTER II

Explosive Rocket with Thrust Augmentation

15. Principle and operation

The system is composed (fig. 6) of the rocket F and an augments tube comprising three principal parts:

A nozzle T' conveying the outside air right up to the exhaust of the rocket nozzle T (section S_1);

A mixer M , the portion of the nozzle at outlet S_2 of which it is assumed that the mixture of rocket gas and the air introduced through the nozzle T' is homogeneous and that the temperature and the speed are sensibly uniform in the straight section S_2 ;

A diffuser D , a nozzle in which the mixture coming from the mixer is reduced in the exhaust section S_e to atmospheric pressure by expansion or compression in the usual manner of gases in nozzles. The preceding scheme can be complicated by multiplying the number of nozzles T' and by making them terminate successively on the axial flow so as to provide in more progressive manner the mixture which must be reached in the central part of the tube called mixer.

The present study is limited to the rocket with simple augmentation as shown in figure 6. It should be an easy matter then to extend it to include the case of the rocket with multiple thrust augmenters tubes.

The thrust augments is a jet device which has the advantage of permitting the entrainment of one fluid by another without involving any movable element; its addition to the explosive rocket gives the latter the characteristics of a propulsion system without moving parts.

At first sight, the increase in volume which the effect of the augments tube must procure and to which there corresponds a reduction in the velocity of the entrained jet seems favorable for increased over-all efficiency, but the fact must not be lost sight of that the effect of the augments, introducing the viscosity and friction which determine the mixing of one of the fluids with the other, necessarily exerts an influence on the thermal efficiency of the system.

An attempt is made further on to illustrate these two principal aspects of the study of the explosive rocket with thrust augmentation.

For the sake of simplification, only the over-all efficiency η_g and the propulsion efficiency η_p are to be considered. It will be recalled that, strictly speaking, it is more rational to consider the efficiencies analogous but defined by a slightly different method, called the efficiency of energy and of reaction as visualized in the foregoing. The more the contribution of the air by the augments increases, the more the difference between the corresponding efficiencies of these two categories are reduced, and this fact, together with the consideration of forward speeds not exceeding 1000 km/h, permits the study of η_g and η_p to be made without trouble.

16. Operating formulas of the rocket with thrust augmentation

(a) Thrust. - Disregarding, as it seems admissible, the aerodynamic resistance R_e of the rocket with thrust augmentation, the thrust is

$$T = mw_e + m'(w_e - V) \quad (42)$$

m and m' denoting the corresponding consumptions (in mass per unit of time) of the powder and air extrained by the augments tube and w_e the corresponding speed of exhaust.

This speed is given by

$$(m + m')w_e^2 = m'V^2 + 2\eta_{th}mL \quad (43)$$

(b) Efficiency. - With η_{th} , η_p , $\eta_g = \eta_{th} \times \eta_p$ denoting, respectively, the thermal, propulsive, and over-all efficiency of the rocket with thrust augmentation defined as in 3, we put

$$\frac{m'}{m} = \mu; \quad \frac{L}{V^2} = q$$

Then the application of (42) and (43) readily gives

$$\eta_p = \frac{1}{\eta_{th} q} \left[\sqrt{(1 + \mu)(\mu + 2\eta_{th} q)} - \mu \right] \quad (44)$$

$$\eta_g = \frac{1}{q} \left[\sqrt{(1 + \mu)(\mu + 2\eta_{th} q)} - \mu \right] \quad (45)$$

To determine the thermal efficiency η_{th} , it is necessary to resort to a theory of gas boosting or augmentation discussed in Appendix I to III.

It assumes the augmentser to be adiabatic, that is, without exchange of heat with the surrounding medium, and compares the burnt gases with the perfect gases, which assumes the presence of condensable water vapor in the gaseous products of the explosion to be practically negligible.

When, in these conditions, Λ and Λ' denote the total heat ($\Lambda = \underline{U} + p\sigma$) per unit mass of burnt gas and air collected by the augmentser tube, functions which for perfect gases depend only the temperature ($d\Lambda = C_d T$, $d\Lambda' = C'_d T$), the thermal efficiency η_{th} of the rocket with thrust augmentation is

$$\eta_{th} = 1 - \frac{\int_{T_a}^{T_e} d\Lambda + \mu d\Lambda'}{L} \quad (46)$$

T_a and T_e signify the temperature of the atmosphere and of the exhaust, assumed uniform in the discharge section S_e (fig. 6).

On denoting the sections of entry and exit of the mixer with subscripts 1 and 2, where the pressure p_1 is assumed constant and uniform, and the state of the gases in the combustion chamber of the rocket with subscript c and that of the air in the atmosphere with subscript a , the total heat Λ and Λ' of the primary and secondary fluid satisfy the relation

$$m\Lambda_2 + m'\Lambda_2' = k \left[m\Lambda_c + m' \left(\Lambda_a' + \frac{v^2}{2} \right) \right] + (1 - k)(m\Lambda_1 + m'\Lambda_1') \quad (47)$$

the coefficient k represents the ratio of the energy $-\Delta C_v$ (taken as absolute value) of viscosity and friction in the mixer to the kinetic

energy $\left(m \frac{w_1^2}{2} + m' \frac{w_1'^2}{2} \right)$ of the fluids at entry in the mixer, which can be expressed by the qualifying relation

$$k = \frac{-\Delta C_v}{m(\Lambda_c - \Lambda_1) + m' \left(\frac{v^2}{2} + \Lambda_a' - \Lambda_1' \right)} \quad (48)$$

Given the pressure $p_c = \lambda_c p_a$ in the combustion chamber of the rocket and the pressure $p_1 = \lambda_1 p_a$ in the mixer, the values of Λ and of Λ' for these pressures can be connected.

In fact, the steady and adiabatic flow in the nozzles T and T' giving access to the mixer is

$$T_c \lambda_1^s = T_1 \lambda_c^s \quad (49)$$

$$T_a \lambda_1^{s'} = T_1' \quad (50)$$

and the flow of the assumedly homogeneous fluid mixture which circulates in the discharge nozzle or diffuser is

$$T_2 \lambda_1^{-s_d} = T_e \quad (51)$$

The exponents s , s' , and s_d in (49), (50), and (51) represent a number which is equal to $\frac{\gamma - 1}{\gamma}$, $\frac{\gamma' - 1}{\gamma'}$, $\frac{\gamma_d - 1}{\gamma_d}$ (γ , γ' , γ_d = the ratio of specific heats at constant pressure and at constant volume of fluid or of the correspondent fluid mixture), if the adiabatic flow in the nozzles under consideration was reversible. The fact that the adiabatic flow in real nozzles is not reversible (due to the internal viscosity of the fluid and the effect of the walls) causes these exponents to be inferior or superior to their value in the case of reversibility, depending upon whether the flow involved is accompanied by expansion or compression.

In nozzle T expansion always prevails ($\lambda_c > \lambda_1$) and

$$s \leq \frac{\gamma - 1}{\gamma}$$

In nozzle T' compression or expansion exists and s' is greater or smaller than $\frac{\gamma' - 1}{\gamma'}$, depending upon whether the mixer operates at positive or negative pressure ($\lambda_1 \gtrless 1$).

Lastly, the flow in the diffuser presents a character opposite to the preceding one.

Any compression existing in the nozzle T' is limited to the value l_1 of ratio λ_1 for which the air collected in the augments tube is immobilized in the mixer inlet

$$l_1 = \left(1 + \frac{v^2}{2C'T_a}\right)^{\frac{1}{s'}} \quad (52)$$

Given T_c , λ_c (the conditions in the combustion chamber) as well as $\mu = \frac{m'}{m}$ (ratio of volumes of the augments tube) and λ_1 (corresponding pressure in the mixer), the equations (46) and (51) can be used, with allowance for ($\lambda_1 < l_1$), to compute the thermal efficiency η_{th} of the rocket with thrust augmentation, provided that the factor k is evaluated in these conditions, which certainly depends on the chosen data.

The theory of gas augments tubes is not far enough advanced yet to permit k to be determined in these conditions; the development of the theory for determining the factor k experimentally calls for systematic variation of the experimental conditions.

However, it is a certainty that k ranges between zero (ideal augments) and unity (augments limited to impossible evacuation). Moreover, it is likely that k increases with the ratio μ of the volumes and with the difference ($w_1 - w_1'$) of the velocities at entrance in the mixer, all other conditions remaining the same.

(c) Intensity of thrust. - As before, the intensity of thrust t of the rocket with thrust augmentation is the ratio of thrust T to the discharge section S_e of the augments tube

$$t = \frac{T}{S_e}$$

With $p_e = p_a$ denoting the pressure at exit from the augments tube and R and R' the Mariotte and Gay-Lussac constants for powder gases and air trapped by the augments tube, while taking into account (42), the intensity of thrust t of the rocket with thrust augmentation is

$$t = \frac{T}{S_e} = p_a \frac{(1 + \mu) w_e^2 - \mu V w_e}{(R + \mu R') T_e} \quad (53)$$

where w_e is given by (43). This formula is to be compared with the pure explosive rocket formula (28).

17. General formulas for comparing the efficiencies of the true rocket and the rocket with thrust augmentation

The problem involved here is to ascertain whether the addition of a thrust augments to an explosive rocket produces certain advantages.

To this end, the rocket with thrust augmentation, for which the equations of operation are to be established, is compared with a true rocket operating with the same consumption m of the explosive of the same nature, at the same pressure $p_c = \lambda_e p_a$ and at the same temperature T_c in the combustion chamber.

The corresponding quantities for the true rocket are distinguished by two dashes; the subscripts denote the corresponding states of combustion and exhaust.

It should be noted, however, that the combustion temperature T_c in both types of rockets is very high with respect to the temperatures indicated by T_e or T_e'' , T_1 , T_1' , and T_a . To simplify matters, the mean specific heat at constant powder gas pressure between T_c and T_a is designated by C_m and the true specific heat of these gases is regarded as quasi-constant and equal to hC_m so that the temperature of these gases remains within the sphere comprising the values T_e , T_e'' , T_1 , T_1' , and T_a . Lastly, it is conceded that within this field the air trapped by the thrust augments has the same specific heat as the powder gases. The factor h seen above is now a little less than unity by reason of the increase in the specific heats of the gases with the temperature.

These approximations, which are suitable for a summary study, are used to write

$$L = C_m [\overline{T_c} - T_a] \quad (54)$$

$$\eta_{th}'' = 1 - h \frac{T_e'' - T_a}{T_c - T_a} \quad (55)$$

$$\eta_{th} = 1 - h(1 + \mu) \frac{T_e - T_a}{T_c - T_a} \quad (56)$$

It is easily verified that this equation (55), corresponding to the true rocket, is identical with formula (26) for this rocket, when

putting $h = 1$, which disregards the variations in specific heat with the temperature, variations previously taken into consideration summarily by introducing the factor h slightly below unity and which represents the ratio of the specific heats of the powder gases (or of the air) at low and high temperatures.

On the other hand, the final temperature T_2 at the mixer outlet of the rocket with thrust augmentation is, by (47)

$$(1 + \mu)T_2 = (1 - k)(T_1 + \mu T_1') + \frac{k}{h} \left[T_c - T_a \left(1 - \frac{\mu}{2A} - h - \mu h \right) \right] \quad (57)$$

the parameter A represents the quantity $C_m T_a / V^2$.

The temperatures T_1 , T_1' , T_e , and T_e'' are linked to the corresponding ratios of compression or expansion by the polytropic equation by which the transformation of the gases in the corresponding nozzles are assumed to be expressed. For the rocket with thrust augmentation, the equations (49), (50), and (51) should be introduced.

For the true rocket

$$T_e = T_c \lambda_c^{-s}$$

By the use of these expressions, the ratio of the thermal efficiencies η_{th} and η_{th}'' evaluated above assumes the form

$$\frac{\eta_{th}}{\eta_{th}''} = 1 - \frac{hC_m}{\eta_{th}'' L} \left\{ (1 - k) T_c \lambda_c^{-s} \lambda_1^{s-s_d} + \frac{k}{h} T_c \lambda_1^{-s_d} - T_c \lambda_c^{-s} - T_a \left[\frac{k}{h} \left(1 - h - \mu h - \frac{\mu}{2A} \right) \lambda_1^{-s_d} + \mu - (1 - k) \mu \lambda_1^{s'-s_d} \right] \right\} \quad (58)$$

In this equation, the exponents s , s' , and s_d signify the average values of the exponents of polytropic expansion or compression which must be chosen as indicated previously.

The expression thus obtained lends itself to a very accurate evaluation of the thermal efficiency ratios of the compared rockets when considering a specific case, but its corresponding complication does not lend itself to a simple study of a general case.

As a result, this expression is simplified by putting $s = s' = s_d$ which assumes that the compressions and expansions in the nozzles (but not in the mixer) are adiabatic and reversible and the ratio $s = \frac{\gamma - 1}{\gamma}$ nearly independent of the temperature.

Equation (58) becomes then

$$\frac{\eta_{th}}{\eta_{th}''} = 1 - k \frac{C_m}{\eta_{th}'' L} \left\{ T_c (\lambda_1^{-s} - h \lambda_c^{-s}) - T_a \left[\left(1 - h - \mu h - \frac{\mu}{2A} \right) \lambda_1^{-s} + \mu h \right] \right\} \quad (59)$$

This equation can be simplified by utilizing equation (48) which defines the coefficient k which is necessarily contained between zero and unity.

By putting, in fact

$$\Delta C_v = -\bar{\omega} m L \quad (60)$$

the factor $\bar{\omega}$, essentially positive, represents the ratio of the negative energy of the effects of viscosity and friction in the mixer of the augments tube to the heat value of the explosive consumed in the same time; hence equation (59) takes the simple form

$$\frac{\eta_{th}}{\eta_{th}''} = 1 - \lambda_1^{-s} \frac{\bar{\omega}}{\eta_{th}''} \quad (61)$$

Now it is readily apparent that the thermal efficiency η_{th} of the rocket with thrust augmentation is always lower than the efficiency η_{th}'' of the corresponding true rocket.

The ratio of these efficiencies for rockets with suitably designed nozzles depends only upon the ratios $\mu = \frac{m'}{m}$ and $\lambda_1 = \frac{p_1}{p_a}$ which characterize the operation of the rocket with thrust augmentation and determine the factors k or $\bar{\omega}$ corresponding to the energy losses which occur in the mixer.

At the present state of knowledge, it is unfortunately impossible to evaluate the coefficient k (or $\bar{\omega}$) corresponding to the mixer of a gas thrust augments as function of its operational conditions μ and λ_1 .

Still, it may be considered likely, if not certain, that k increases either when the ratio μ of the volume of entrained gas and of the moving gas increases, or when the difference $(w_1 - w_1')$ in the respective speeds of these fluids at entrance in the mixer increases in absolute value. On the other hand, the possible values between zero and unity in any specific case of coefficient k can be computed in probable manner.

A further attempt to effect the comparison of the rocket with thrust augmentation with the pure rocket is described later on.

As essential term for such a comparison, it is expedient to take the over-all efficiency ratios η_g and η_g'' of these rockets.

These efficiencies are expressed by (45) and (22); hence

$$\eta_g = \frac{1}{q} \left[\sqrt{(1 + \mu)(\mu + 2\eta_{th}q)} - \mu \right]$$

$$\eta_g'' = \frac{1}{q} \sqrt{2\eta_{th}''q}$$

and the ratio of these efficiencies is

$$\frac{\eta_g}{\eta_g''} = \frac{\sqrt{(1 + \mu)(\mu + 2\eta_{th}q)} - \mu}{\sqrt{2\eta_{th}''q}} \quad (62)$$

To ascertain whether the over-all efficiency of the rocket with thrust augmentation is perhaps superior to that of the true rocket, that is, whether the preceding ratio is perhaps greater than unity, it is necessary and it suffices, according to (62), that

$$\frac{\eta_{th}}{\eta_{th}''} > \frac{1}{1 + \mu} \left[1 + \frac{\mu}{2\eta_{th}''q} \left(2\sqrt{2\eta_{th}''q} - 1 \right) \right] \quad (63)$$

It is known from (61) that the ratio η_{th}/η_{th}'' is necessarily less than unity and it is easily verified that the second term of (63) represents a quantity less than unity. In consequence, it still does not prohibit contemplating that the rocket with thrust augmentation could have a thermal efficiency η_{th} such that, though below the η_{th}'' of

the corresponding true rocket, the inequality (63) might be satisfied and that, therefore, it might yield

$$\eta_g > \eta_g''$$

This is the question which is now to be analyzed thoroughly by simplifying again the equation (59) of the thermal efficiency ratios of the two rockets.

18. Approximate analysis of the conditions of superiority of the rocket with thrust augmentation

Since the discussion is confined to the comparison of η_{th} and η_{th}'' , it is admitted that h might be put = 1 in formula (59), that is, the change of specific heat with temperature may be disregarded, since the covolume of the powder gases and the difference in the corresponding specific heats of these gases and the air have been ignored.

These approximations are indubitably legitimate considering that only the comparison of the rocket with thrust augmentation and the corresponding true rocket is involved.

The comparison is made by first visualizing the particular case of the rocket with thrust augmentation with mixer operating at atmospheric pressure ($\lambda_1 = 1$) and studying its possible efficiency in relation to the ratio μ of the volume of the thrust augments, then by proceeding to the general case (that is, $\lambda \neq 1$), and comparing it with the first for an identical value of μ .

19. First specific case - mixer at atmospheric pressure ($\lambda_1 = 1$)

In this case, the rocket, operates strictly speaking, the same way in the rocket with thrust augmentation and in the corresponding true rocket, and the question is whether the addition of thrust augmentation which does not change the operation of the rocket affords an increase in over-all efficiency of the whole system.

Adopting the subscript 1 for the characteristic quantities of this particular case, so as to reproduce the condition $\lambda_1 = 1$, we get, with the approximations specified above,

$$\left(\frac{\eta_{th}}{\eta_{th}''}\right)_1 = 1 - k_1 \left[1 + \frac{\mu}{2\eta_{th}'' q} \right]$$

Putting, for simplification⁷

$$a_1 = \sqrt{2\eta_{th}'' q} = \sqrt{2\eta_{th}'' \frac{L}{V^2}} \quad (64)$$

the preceding formula becomes

$$Y_1 = \left(\frac{\eta_{th}}{\eta_{th}''} \right)_1 = \left[1 - k_1 \left(1 + \frac{\mu}{a_1^2} \right) \right] \quad (65)$$

The ratio of over-all efficiencies is

$$X_1 = \left(\frac{\eta_g}{\eta_g''} \right)_1 = \frac{1}{a_1} \left[\sqrt{(1 + \mu)(\mu + a_1^2)(1 - k_1)} - \mu \right] \quad (66)$$

It is a question of knowing, a_1 being given, whether μ values can be realized such that X_1 is greater than unity ($X_1 > 1$) and eventually, to determine the maximum of X_1 .

In this problem, the coefficient k_1 of the kinetic-energy losses in the thrust augments, defined by (48) and corresponding to $\lambda_1 = 1$, is a simple function of μ which depends only upon the velocity and the temperature of the fluids at entry in the mixer, that is, T_e'' , w_e'' , T_a , and V .

When these conditions are fixed, for example, by T_a , V , p_c , and η_{th}'' , the coefficient k_1 is a function determined by μ , which is certainly increasing with μ , but on the subject of which, for lack of being able to take up the theoretical study (cf. appendix III), or being able to refer to systematic tests which have not yet been made, only more or less plausible hypotheses can be made.

In any case, an attempt is made to determine the necessary and sufficient condition at which the ratio X_1 could exceed unity.

⁷The parameter a_1 previously defined by (64) represents the ratio of the rate of ejection $w_e'' = \sqrt{2\eta_{th}'' L}$ of the true rocket at forward speed V . The importance of the speed ratio in all questions of jet propulsion explains the fundamental part played by the parameter a_1 .

This condition can be expressed, by virtue of (66), by

$$k_1 < K_1 = \frac{\mu(1 - a_1)^2}{(1 + \mu)(\mu + a_1^2)}$$

The family of curves $K_1(\mu)$ corresponding to different values of a_1 can be plotted on a diagram.

The values of this function are given in table IV.

Table IV

Value of Function $K_1(\mu)$ and the Maximum of This
Function for Different Values of a_1

$a_1 = \sqrt{2\eta_{th}} q$	Value of K_1 for			Maximum $K_1 =$	For $\mu = a_1 =$
	$\mu = 0$	$\mu = 1$	$\mu = \infty$		
0	1	0.5	0	1	0
.1	0	.4	0	.67	.1
.25	0	.264	0	.36	.25
.5	0	.10	0	.11	.5
1	0	0	0	0	1
2	0	.10	0	.11	2
4	0	.264	0	.36	4
10	0	.4	0	.67	10
∞	0	.5	1	1	∞

The corresponding curves are represented in the diagram of figure 7, along with the position of the maximum of $K_1(\mu, a_1)$. The corresponding curve $K_1(\mu)$ for any value of a_1 is obtained by interpolation, when observing that the maximum of this curve is located at the preceding place and has $\mu = a_1$ for abscissa.

For the rocket with powder B ($L = 5,000,000$) whose thermal efficiency η as true rocket may be supposed to be of the order of 0.4 to 0.6, the previous values indicated by the parameter $a_1 = \sqrt{2\eta_{th}} L/V^2$ correspond to the following values of the forward speed V :

Values of a_1	0	0.1	0.25	0.5	1	2	4	10	∞
$V(\text{m/sec}) \left\{ \begin{array}{l} p^r_{\eta_{th}} = 0.4 \\ p^r_{\eta_{th}} = 0.6 \end{array} \right.$	∞	20,000	8,000	4,000	2,000	1,000	500	200	0
	∞	24,500	9,800	4,900	2,450	1,225	612	245	0

The scope of practical interest being confined to speeds below 200 or 300 m/sec at the most,⁸ it is seen that the only pertinent a_1 values are those above unity, and consequently the curves $K_1(\mu)$ to be retained in figure 7 are those curves situated below the hyperbolic arc corresponding to $a_1 = \infty$ (or $V = 0$) and whose maximum, which has the value $\left(\frac{1 - a_1}{1 + a_1} \right)^2$ and corresponds to the abscissa $\mu = a_1$, is found to be

so much higher and shifted so much farther toward the right as a_1 is greater, that is, as the speed V is slower. When the curve $k_1(\mu)$ is plotted (which for the time being must be made by way of assumption or by guess), the ratio X_1 of the over-all efficiencies can exceed unity only at values μ for which the curve k_1 is located below the curve $K_1(\mu)$ which corresponds to the value of a_1 in question.

The maximum of the ratio X_1 occurs at

$$\frac{d}{d\mu} \left[\sqrt{(1 + \mu)(\mu + a_1^2)(1 - k_1)} - \mu \right] = 0$$

that is, at the point on curve $k_1(\mu)$ where the latter is tangential to a curve of the family

$$\sqrt{(1 + \mu)(\mu + a_1^2)(1 - k_1)} = \mu + a$$

a designating an arbitrary constant.

The parameter a_1 being fixed, the ordinate K_1' of the curves of this family can be written

$$K_1' = K_1 - \frac{(a - a_1)(a + a_1 + 2\mu)}{\mu(1 - a_1)^2} \quad (68)$$

⁸For ballistic applications of rockets with thrust augmentation, it is evident that much higher speeds can be visualized, but in this instance the present or future aviation is of sole interest.

and the curves of this family situated below curve K_1 are obtained by giving to the arbitrary constant a some values greater than a_1 .

By way of example, figures 8 and 9 represent some of these curves as well as the corresponding curve K_1 for two particular values of a_1 ($a_1 = 20$ and $a_1 = 10$).

These values correspond, on a rocket with powder B ($L = 5,000,000$) with a thermal efficiency η_{th} as pure rocket assumed equal to 0.50, to the following values and speeds V :

$$V = 112 \text{ m/sec} = 402 \text{ km/h, for } a_1 = 20$$

$$V = 224 \text{ m/sec} = 804 \text{ km/h, for } a_1 = 10$$

Consider, by way of example, the diagram of figure 8. On bearing in mind that, in order to be able to plot the curve $k_1(\mu)$ corresponding to the thrust augmentser involved, the eventual point of contact of this curve with one of the curves of the family K_1' must be known, and consequently also the volume μ of the thrust augmentser which gives the maximum superiority of the rocket with thrust augmentation over the corresponding plain rocket under the particular conditions.

But, as already pointed out, the present state of knowledge on gas thrust augmentsers affords no accurate information about the behavior of curve $k_1(\mu)$; hence the necessity of risking intuitive hypotheses about this behavior, while stressing the fundamental interest which attaches to the systematic study of the function $k_1(\mu)$ for different types of thrust augmentsers. The curve $k_1(\mu)$ which starts from zero for $\mu = 0$ is likely to be an ascending curve when μ increases and also when its curvature has a constant sign, hence, when it is of one of the types (I), (II), or (III) represented in figure 10, depending upon whether its curvature is turned upward or downward. The operation of the thrust augmentser (at least according to the conception of perfect or homogeneous mixture adopted in the ordinary theory) being assured by the actions of the friction or the viscosity, we are inclined to believe that these phenomena permit the entrainment of only a finite relative volume with the losses which undoubtedly increase quite rapidly as the limit of the volume is reached.

Following this point of view, the curve $k_1(\mu)$ of type (I) of figure 10 was provisionally adopted.

Now an attempt is made to compute the possible effect of such a characterized thrust augmentser in the case of propulsion corresponding to, say, $a_1 = 20$.

Draw the curve K_1 of figure 8 on the diagram of figure 11, and then a curve C_1 supposed to represent the effect of the thrust augments. The powder gases arrive at the mixer inlet with a speed $w_1 = w_e'' = 2240$ m/sec and at an absolute temperature T_e'' equal to⁹ $1369^\circ = 273^\circ + 1096^\circ$, the rocket being assumed to use powder B and operate with a thermal efficiency (as pure rocket) equal to 0.50 in a standard atmosphere ($T_a = 273^\circ + 15 = 288^\circ$). At the same time the entrained air arrives with a speed w_1' equal to the forward speed $V = 112$ m/sec, and at a temperature equal to the surrounding temperature $T_a = 288^\circ$.

The hypothetical curve C_1 indicates that, in these conditions, the volume of entrained air cannot exceed 13.5 times that of the motive fluid. This curve touches the family of curves K_1' of figure 8 at a point M_1 in figure 11, of the abscissa $\mu = 6$ where k_1 assumes the value 0.3. It is for this volume that the thrust augments involved should attain its highest superiority over the corresponding plain rocket. It is easily verified by computing the thermal efficiency ratio X_1 for different points of the curve situated below curve k_1 by equation (66).

This method affords the data given in table V as well as the values of the thermal efficiency ratios Y_1 computed by (65) and the proper efficiency values of the visualized rocket with thrust augmentation, the efficiencies of the plain rocket taken for example being

$$\eta_{th}'' = 0.50 \quad \eta_g'' = 0.05$$

Thus in the preceding hypothesis, the over-all efficiency can be sensibly doubled by the effect of thrust augmentation (in spite of the 30-percent drop in thermal efficiency). It passes from the obviously insufficient value of 0.05, to about 0.10 and approaches interesting values.

⁹The value of T_e'' is, like those indicated in table II, obtained by neglecting the covolume of the gases of the powder and the change of their specific heat with the temperature. The error introduced has evidently no significance for the purely speculative considerations exposed above.

Table V

Rocket with Thrust Augmentation Characterized

by Curve C_1 of Figure 11

	Values of:					Observations
	k_1	$X_1 = \left(\frac{\eta_g}{\eta_g''}\right)_1$	$Y_1 = \left(\frac{\eta_{th}}{\eta_{th}''}\right)_1$	$(\eta_g)_1$	$(\eta_{th})_1$	
$\mu = 3$	0.13	1.72	0.869	0.086	0.434	Assumed maximum of X_1
6	.30	1.92	.696	.096	.348	
10	.60	1.62	.385	.081	.192	

Now the fictitious curve C_1 representing the function $k_1(\mu)$ is replaced by the more optimistic curve C_2 , also shown in figure 11 and for which the limit of the corresponding entrained volume reaches 25 instead of 13.5. The optimum operation of the thus-characterized rocket with thrust augmentation corresponds to point M_2 where curve C_2 touches K_1' or at $\mu = 13.5$ and $k_1 = 0.33$.

Moreover, table VI, which gives, with the aid of curve C_2 and formulas (65) and (66), the characteristic elements of the comparison for different μ , can be prepared.

Table VI

Rocket with Thrust Augmentation Characterized

by Curve C_2 of Figure 11

	Values of:					Observations
	k_1	X_1	Y_1	$(\eta_g)_1$	$(\eta_{th})_1$	
$\mu = 10$	0.23	2.45	0.764	0.122	0.382	Assumed maximum of X_1
13.5	.33	2.5	.659	.125	.329	
19	.55	2.12	.424	.106	.212	

These hypothetical conditions, which are more optimistic than the preceding ones, afford an over-all efficiency 2.5 times higher than that of the plain rocket. Next we compute in the same manner the possible benefit by assuming the same rocket propelled at twice the

speed, that is, $a_1 = 10$, the case which corresponds to the diagram of figure 9. In this case curve $k_1(\mu)$ will obviously be below and to the right of that which corresponds to the preceding case, since, the temperature conditions not being modified at entry in the mixer, the difference in jet velocity, which causes the entrainment of one by the other at the expense of a kinetic energy loss characterized by k_1 , is reduced from $2240 - 112 = 2128$ m/sec to $2240 - 224 = 2016$ m/sec. However, the curve C_1 of figure 11 is retained for comparison with the curves of figure 9 which correspond to the case of $a_1 = 10$. It affords the disposition represented in figure 12 for which the best operation of the rocket with thrust augmentation corresponds to point M_1 , that is, to $\mu = 5.3$ and $k_1 = 0.25$. In this case, where the plain rocket has a thermal efficiency of $\eta_{th} = 0.50$ and an over-all efficiency of $\eta_g = 0.10$, the characteristic elements of the comparison are summarized in table VII.

Table VII

Rocket With Thrust Augmentation Characterized
by Curve C_1 of Figure 12

	Values of:					Observations
	k_1	X_1	Y_1	$(\eta_g)_1$	$(\eta_{th})_1$	
$\mu = 3$	0.13	1.592	0.866	0.159	0.433	Assumed maximum of X_1
5.3,	.25	1.70	.736	.170	.368	
10	.6	1.50	.34	.120	.17	

When the curve C_1 is replaced by C_2 , a more optimistic estimation of the function k_1 , (less optimistic, however, than in the case of $a_1 = 20$), the figure 12 shows that the best operation of the rocket with thrust augmentation corresponds to the point M_2 where $\mu = 8.8$ and $k_1 = 0.18$. The characteristic elements of the comparison for different values of μ are indicated in table VIII.

Table VIII

Rocket with Thrust Augmentation Characterized

by Curve C_2 of Figure 12

	Values of:					Observations
	k_1	X_1	Y_1	$(\eta_g)_1$	$(\eta_{th})_1$	
$\mu = 6$	0.12	1.96	0.873	0.196	0.436	Assumed maximum of X_1
8.8	.18	2.08	.804	.208	.402	
15	.38	1.88	.563	.188	.281	

On reconciling the data of tables VII and VIII for $a_1 = 10$ with those of tables V and VI for $a_1 = 20$, it is seen that, when a_1 passes from 20 to 10, that is, when the speed V passes from 402 to 804 km/h, the thermal efficiency η_{th} of the plain rocket being assumed equal to 50 percent, the maximum ratio of the over-all efficiencies of the rocket with thrust augmentation to the plain rocket decreases a little for the comparable hypotheses of the possible variation of $k_1(\mu)$ which characterizes the mixer of the thrust augments. This maximum ratio shifts from 1.92 to 1.70 or from 2.5 to 2.08, depending upon whether k_1 is given the shape of curve C_1 or C_2 in figures 11 and 12.

In the second case ($a_1 = 10$ or $V = 804$ km/h), the over-all efficiency of the rocket with thrust augmentation thus reaches at the most and by virtue of the doubled efficiency of the pure reference rocket values of the order of 0.17 to 0.208, depending on whether k_1 follows one or the other of the hypotheses C_1 or C_2 .

It must therefore be concluded that, if these hypotheses are realizable, the rocket with thrust augmentation is, at these high forward speeds, susceptible to an over-all efficiency comparable to that obtained either with the orthodox engine-propeller unit at low speeds or with the pure rocket at considerably higher speeds.

The accuracy of this result depends only, it is pointed out, on the possibility of securing a sufficiently efficient mixer, and this can be established only by appropriate experimental research.

A few more data are added to determine exactly what can be accomplished by a rocket with optimum thrust augmentation corresponding to the conditions of table VII, that is, to the point M_1 of figure 12:

$$a_1 = 10 \quad \mu = 5.3 \quad k_1 = 0.25 \quad (\eta_g)_1 = 0.170 \quad (\eta_{th})_1 = 0.368$$

In this particular case, the temperature T_e of the gases at the thrust augments exit is (according to (56) simplified by $h = 1$ and where $T_c = 2450^\circ$) equal to $273^\circ + 231^\circ = 504^\circ$ instead of $273^\circ + 1096^\circ = 1369^\circ$ for the powder gases at entrance in the mixer.

The speed of ejection of the gases w_e at the exit of the thrust augments is 925 m/sec at a propulsive velocity of 224 m/h.

The intensity of thrust t (thrust per unit of sectional area S_e of the thrust augments outlet) is, according to (53), where $R = R'$, equal to 2.25 times the pressure p_a of the outside atmosphere at a temperature of the latter equal to 288° .

Lastly, the air intake section S_a for the thrust augments referred to the evacuation section S_e of the thrust augments is

$$\frac{S_a}{S_e} = \frac{\mu}{1 + \mu} \frac{T_a}{T_e} \frac{w_e}{V} = 1.98$$

In other words, the part upstream from the thrust augments has a frontal section about twice as great as the part downstream.

20. Second general case - effect of pressure p_1 at the mixer

The pressure p_1 at the mixer of the thrust augments being defined by the ratio $\lambda_1 = p_1/p_a$, it may be asked whether it is important to increase the over-all efficiency η_g of the thrust augments and to increase or decrease λ_1 for a specified corresponding volume $\mu = m'/m$ and, eventually, what value this characteristic ratio should have to ensure the maximum of η_g .

In other words, η_g being, for the conditions of combustion (T_c, p_c), a specific fuel and atmosphere, solely a function of the operating conditions λ_1 and μ of the thrust augments, it means finding the values of the parameters that give the highest possible η_g , the hypothetical variation of which had been previously studied as function of μ for the particular case $\lambda_1 = 1$.

Retaining the approximations of the preceding article, the partial derivative of η_g with respect to λ_1 (μ being fixed) is analyzed.

By derivation of (62) with respect to λ_1

$$Z = \frac{\partial \eta_g}{\partial \lambda_1} = \sqrt{\frac{1 + \mu}{\mu + 2\eta_{th}q}} \frac{\partial \eta_{th}}{\partial \lambda_1} \quad (69)$$

and the function Z under consideration has the same sign as the function

$$W(\lambda_1, \mu) = \frac{\partial \eta_{th}}{\partial \lambda_1}$$

which is now being considered.

By equation (59), where for simplification $h = 1$

$$W = - \frac{C_m}{\eta_{th}''L} \left\{ T_c (\lambda_1^{-s} - \lambda_c^{-s}) - \mu T_a \left[1 - \left(1 + \frac{1}{2A} \right) \lambda_1^{-s} \right] \right\} \frac{\partial k}{\partial \lambda_1} +$$

$$\frac{skC_m}{\eta_{th}''L} \lambda_1^{-s-1} \left[T_c + \mu T_a \left(1 + \frac{1}{2A} \right) \right] \quad (70)$$

or with (59) taken into account

$$W = \frac{(\eta_{th}'' - \eta_{th})}{k\eta_{th}''} \frac{\partial k}{\partial \lambda_1} + \frac{skC_m \lambda_1^{-s-1}}{\eta_{th}''L} \left[T_c + \mu T_a \left(1 + \frac{1}{2A} \right) \right] \quad (71)$$

The formula shows that W is the sum of two terms, the second of which is essentially positive and the first of contrary sign to $\frac{\partial k}{\partial \lambda_1}$.

Therefore, when $\frac{\partial k}{\partial \lambda_1}$ is negative, W is essentially positive and η_g is increased for a fixed corresponding volume μ by increasing λ_1 , that is, the pressure in the mixer. It will be recalled that λ_1 can in no case exceed the limit value

$$\lambda_1 = \left(1 + \frac{v^2}{2C'T_a} \right)^{\frac{1}{s'}} \quad (52)$$

As for the variation of the function k_1 , as in function of μ , the variation of k for $\lambda_1 = 1$ can be effected only by way of an assumption. The function k defined by (48) and which characterizes the losses of kinetic energy in the mixer, depends, for an optimum mixer, only on the conditions $(p_1, T_1, T_1', w_1, w_1')$ at entry in the said mixer and the corresponding volume of μ .

Given the nature of the fluids, the conditions (p_c, T_c) in the combustion chamber of the rocket and the outside temperature, the factor K for the best mixer depends only on λ_1 and μ .

In first approximation, it may be admitted that, for a specified μ and constant temperature in the mixer inlet, k increases with the difference in the jet velocities $(w_1 - w_1')$ entering the mixer, the difference involving the friction and the viscosity, and so enables the mixer to accomplish its function.

A change λ_1 modifies both the jet velocities $(w_1 - w_1')$ and the temperatures T_1 and T_1' of the fluids at the mixer inlet.

Suppose that the effect of the variation of $(w_1 - w_1')$ on k is preponderate. In that event it may be admitted that $\partial k / \partial \lambda_1$ has the same sign as $d(w_1 - w_1') / d\lambda_1$; this derivative is expressed as

$$w_1^2 = 2C_m T_c (1 - \lambda_c^{-s} \lambda_1^s)$$

$$w_1'^2 = V^2 + 2C_m T_a (1 - \lambda_1^s)$$

whence, by utilizing the notation $q = L/V^2$, while putting $B = L/C_m T_a$ and bearing in mind that $\eta_{th}'' L = C_m T_c (1 - \lambda_c^{-s})$, $L = C_m (T_c - T_a)$

$$\frac{d(w_1 - w_1')}{d\lambda_1} = s\lambda_1^{s-1} \sqrt{\frac{C_m T_a}{2}}$$

$$\left[\frac{\sqrt{(1+B) - \lambda_1^s [1 + B(1 - \eta_{th}'')]}}{\sqrt{1+B - \lambda_1^s [1 + B(1 - \eta_{th}'')]}} - \frac{[1 + B(1 - \eta_{th}'')] \sqrt{1 - \lambda_1^s + \frac{B}{2q}}}{\sqrt{1 - \lambda_1^s + \frac{B}{2q}}} \right] \quad (72)$$

This formula and consequently also $\frac{\partial k}{\partial \lambda_1}$ cancel out for the value l_2 of λ_1 so that

$$l_2 = \left\{ \frac{\left[1 + B(1 - \eta_{th}'')\right]^2 \left[1 + \frac{B}{2q}\right] - (1 + B)}{B(1 - \eta_{th}'') \left[1 + B(1 - \eta_{th}'')\right]} \right\}^{\frac{1}{s}} \quad (73)$$

In this formula, it is advisable to give the exponent s a value corresponding to the sufficiently low temperatures prevailing in the mixer, especially when l_2 is less than unity. Choosing the value $s = 0.286$ (which corresponds to the reversible adiabatic transformations upstream from the mixer with a specific heat ratio of $\gamma = 1.4$) affords the table IX for the values l_2 of a rocket with thrust-augmentation using powder B and for several thermal efficiency values η_{th}'' of the pure reference rocket and at different jet velocities V (at standard atmosphere, that is, $T_a = 273 + 15 = 288^\circ$).

Table IX

Values of l_2 for the Rocket with
Thrust-Augmentation Powder B

$\eta_{th}'' =$	0.2	0.4	0.6
m/sec			
$V = 0$	0.852	0.637	0.191
100	.901	.659	.203
200	1.0	.737	.238
300	1.154	.880	.312

The variation of l_2 as function of V for different η_{th}'' is plotted in figure 13. This diagram shows that, up to very high speeds of propulsion V and except for very low η_{th}'' of the reference rocket, quantity l_2 is less than unity. It is also possible to compute the upper limit of the speed of propulsion V above which, for given η_{th}'' and T_a , the quantity l_2 is greater than unity. This

speed V_2 , derived from (73), where $q = L/V^2$ and $l_2 = 1$, is given by the formula

$$V_2 = \sqrt{\frac{2\eta_{th}''L}{1 + B(1 - \eta_{th}'')}} \quad (74)$$

and indicated in table X, and plotted in figure 14, for two values of T_a corresponding to standard atmosphere 0 and 6000m.

Table X

Speed Limit V_2 for which $l_2 = 1$

Rocket with Powder B

$\eta_{th}'' =$	0.2	0.3	0.4	0.5	0.6
$V_2 = \begin{cases} \text{for } T_a = 288^\circ \\ \text{(standard sea level)} \end{cases}$	202	277	364	471	614
$\text{m/sec } \begin{cases} \text{for } T_a = 249^\circ \\ \text{(6000m)} \end{cases}$	177	245	322	419	549

It is readily apparent that for acceptable η_{th}'' values (ranging between around 0.4 and 0.6) and at speeds below $V = 1000$ km/h at least, that is, in the range of application in question, the quantity l_2 is certainly less than unity.

From this it is concluded that

(a) Function $k(\lambda_1, \mu)$ is representable by a set of curves analogous to that of figure 15.

(b) The point of optimum operation (λ_1, μ) where the over-all efficiency η_g of the rocket with thrust augmentation attains its highest maximum lies between the ordinates $\lambda_1 = l_2$ and $\lambda_1 = l_1$.

The location of this optimum point is predicated upon the knowledge of $k(\lambda_1, \mu)$, on the subject of which the lack of insufficient theoretical, as well as experimental information, has already been noted. However, proceeding from the value $\lambda_1 = 1$ for which the variation in over-all efficiency η_g had been studied as function of μ according to certain assumptions regarding function $k_1(\mu) = k(\lambda_1 = 1, \mu)$, it should be

interesting to know whether η_g improves or worsens by increasing λ_1 , while the parameter μ is assumed fixed. This involves studying the function $W(\lambda_1, \mu)$ expressed by (71) and making $\lambda_1 = 1$. This function then becomes

$$W_1(\mu) = -\left(1 + \frac{\mu}{2\eta_{th}''q}\right)\left(\frac{\partial k}{\partial \lambda_1}\right)_{\lambda_1=1} + \frac{sk_1}{\eta_{th}''} \left[\frac{T_c + \mu T_a \left(1 + \frac{1}{2A}\right)}{T_c - T_a} \right] \quad (75)$$

In normal conditions, that is, when λ_2 is less than unity (case of figure 15), $\left(\frac{\partial k}{\partial \lambda_1}\right)_{\lambda_1=1}$ is a function of μ which is 0, for $\mu = 0$ and constantly increases with μ up to the limit at which k reaches its maximum equal to unity; but k_1 is also (cf. fig. 10) a function which presents the same characteristics. Hence, it follows that, according to (75), W_1 is the difference of the two functions of μ , simultaneously zero for $\mu = 0$ and increasing with μ . At $\mu = 0$, W_1 is zero and the value of λ_1 does not affect the over-all efficiency when the augments tube is practically nonexistent. When μ increases, the sign and the value of W_1 depend upon the corresponding course of the two positive terms of which W_1 constitutes the difference. This corresponding course is actually too little known, (not to say entirely unknown), to make it possible to bring the preceding considerations to likely conclusions or to numerical interpretations.

B. ORDINARY FUEL ROCKETS

Chapter I - True Rocket with Ordinary Fuel

21. Definition

This type of rocket is illustrated in figure 16. It comprises a generator G of burnt-gases feeding an expansion nozzle.

The generator draws the fuel through the pipe line C from tank R located outside and the air required for combustion from the outside atmosphere through an orifice A of the air intake, facing upstream. The expansion nozzle evacuates in the outside medium through the evacuation orifice E located at the rear end of the rocket. The fuel may be solid, liquid, or gaseous, but for the present, the discussion is limited to liquid fuels such as are obtained by the distillation of kerosene or coal and which are in every respect the most interesting for application in aviation by reason of their great heat value, of the order of 10,000 to 11,000 cal/kg, which permit the lowest consumption by weight and

hence the greatest range. In order that the nozzle expand the burnt gases effectively with increasing velocity (starting from an initial state without velocity) it is necessary that the combustion of the fuel be effected in the generator at a pressure higher than the surrounding pressure and, consequently, that the fuel and the combustion air be compressed suitably in this generator before combustion, unless the combustion itself is effected with increased pressure, as, for example, at constant volume. Thus, the generator constitutes a heat engine with internal combustion characterized by the fact that it furnishes no mechanical energy utilizable on the outside, but solely burnt and compressed gases. The essential importance that attaches to the increase in the thermal efficiency of the rocket stipulates, as for all heat engines, that this compression be effected as much as possible before combustion, whatever the method of combustion used (constant volume, constant pressure or otherwise). The corresponding compressor or compressors absorb energy which must be supplied by a part of the generator under the action of all or part of the burnt gases, the part that constitutes an engine furnishing exactly the entrainment energy of the compressor or compressors. Different types of generators, more or less complex, can thus be visualized, composed of elements connected more or less to the constitutive elements of known heat engines. For the moment, it is assumed that the products ejected by the rocket constitute a homogeneous mixture moving at uniform exhaust velocity w_e in the exhaust section.

22. Rocket operation formulas

Using the same notation as for the explosive rocket, a_0 indicates the mass of air required for the complete combustion per unit mass of fuel; a , the mass of air effectively consumed by the rocket per unit mass of fuel; $\alpha = \frac{a}{a_0} - 1$, the "dilution" of the combustible mixture utilized in the rocket.

The rocket consuming m mass units of fuel per unit time, the corresponding volume of consumed air is

$$ma = ma_0(1 + \alpha) \quad (76)$$

and the total volume of ejected gases is

$$m(1 + a) = m[1 + a_0(1 + \alpha)]$$

According to the definitions adopted at the start of the present report and by disregarding again the outside aerodynamic resistance of the rocket conceived as an isolated system of propulsion, the following expressions can be immediately established:

(a) The corresponding speed of exhaust w_e is

$$w_e = V \sqrt{\frac{2\eta_{th}L}{(1+a)V^2} + \frac{a}{1+a}} \quad (77)$$

L denotes the lower heat value at constant pressure per unit mass of fuel, to be expressed in units of energy. In the M.K.S. system, L is represented, for a fuel of 11,000 cal/kg, by $11,000 \times 9.81 \times 425 = 45,900,000$, as compared to 2,760,000 and 5,000,000 attributed to black powder and powder B, respectively.

Equation (77) shows that, η_{th} being assumed positive, the speed is always positive and that the latter is always greater than V as long as

$$2\eta_{th}L > V^2$$

(b) The thrust T is given by the formula

$$T = m \left[(1+a)w_e - aV \right] = maV \left[\sqrt{\left(\frac{1+a}{a} \right) \left(1 + \frac{2\eta_{th}L}{aV^2} \right)} - 1 \right] \quad (78)$$

(c) Intensity of thrust t , that is, the traction referred to the exhaust section S_e of the ejected gases, is given by the formula

$$\begin{aligned} t = \frac{T}{S_e} &= p_a \frac{V^2}{RT_e} \left[\left(\frac{w_e}{V} \right)^2 - \left(\frac{a}{1+a} \right) \left(\frac{w_e}{V} \right) \right] \\ &= p_a \frac{V^2}{RT_e} \left[\frac{2\eta_{th}L}{(1+a)V^2} + \frac{a}{1+a} \left\{ 1 - \sqrt{\frac{2\eta_{th}L}{(1+a)V^2} + \frac{a}{1+a}} \right\} \right] \quad (79) \end{aligned}$$

R denotes the Mariotte and Gay-Lussac constant per unit mass of burnt gases assumed comparable to perfect gases ($p = \rho RT$) and T_e is the exhaust temperature.

For complete combustion and completely adiabatic rocket (generator + nozzle)

$$RT_e = \bar{R}T_a + \left(\frac{\gamma - 1}{\gamma} \right) \frac{\eta_{th}L}{1+a} \quad (80)$$

γ is the ratio of specific heats of the burnt gases (average value between T_e and T_a in this ratio).

(d) The propulsive efficiency η_p of the rocket is according to the qualifying equation (9)

$$\eta_p = \frac{TV}{m\eta_{th}L} = \frac{aV^2}{\eta_{th}L} \left[\sqrt{\left(\frac{1+a}{a}\right) \left(1 + \frac{2\eta_{th}L}{aV^2}\right)} - 1 \right] \quad (81)$$

(e) The over-all efficiency η_g of the rocket is according to the qualifying equation (8)

$$\eta_g = \eta_{th}\eta_p = \frac{aV^2}{L} \left[\sqrt{\left(\frac{1+a}{a}\right) \left(1 + \frac{2\eta_{th}L}{aV^2}\right)} - 1 \right] \quad (82)$$

23. Note - case of the explosive rocket

In order to obtain again the case of the true explosive rocket, it is sufficient to nullify the corresponding air consumption a of the rocket considered here. In this event the formulas of article (7) become

$$T = m\sqrt{2\eta_{th}L} \quad (20)$$

$$\eta_p = V\sqrt{\frac{2}{\eta_{th}L}} \quad (21)$$

$$\eta_g = V\sqrt{\frac{2\eta_{th}}{L}} \quad (22)$$

24. Simplification of the preceding formulas

In the foregoing formulas, the ratio w_e/V plays a capital part. Consider the effect of the ratio $(1+a)/a$ on the latter. For the combustion involved, the proportion of air required for complete combustion is, theoretically, at least of from 15 to 16, and, for practical purposes, of from 20 to 25. The advantage of raising η_g by increasing the corresponding volume $(1+a)$ of the burnt gases and consequently the proportion of air a , except when it lowers the thermal efficiency η_{th} too much, prompts the consideration of values of a of the order of 25 to 30 at least. The ratio $\left(\frac{1+a}{a}\right)$ which tends toward unity when a increases differs therefore by 3 to 4 percent at the most.

Formula (77) can be written

$$\frac{w_e}{V} = \sqrt{\frac{a}{1+a}} \sqrt{1 + 2 \frac{\eta_{th} L}{a V^2}}$$

The replacement of $\frac{a}{1+a}$ by unity introduces a corresponding error of no more than 1 to 2 percent, which is regarded as being entirely negligible in the present study. Replacing, accordingly, the ratio $\frac{1+a}{a}$ by unity, formulas (77) to (82) become, after making $L/V^2 = q$

$$\frac{w_e}{V} = \sqrt{1 + \frac{2\eta_{th} q}{a}} \quad (83)$$

$$T = maV \left[\sqrt{1 + \frac{2\eta_{th} q}{a}} - 1 \right] \quad (84)$$

$$t = p_a \frac{V^2}{RT_e} \left[1 + \frac{2\eta_{th} q}{a} - \sqrt{1 + \frac{2\eta_{th} q}{a}} \right] \quad (85)$$

$$RT_e = RT_a + \left(\frac{\gamma - 1}{\gamma} \right) \frac{\eta_{th} q}{a} V^2 \quad (86)$$

$$\eta_p = \frac{a}{\eta_{th} q} \left[\sqrt{1 + \frac{2\eta_{th} q}{a}} - 1 \right] \quad (87)$$

$$\eta_g = \frac{a}{q} \left[\sqrt{1 + \frac{2\eta_{th} q}{a}} - 1 \right] \quad (88)$$

In these equations, the nondimensional quantity

$$Q = \frac{\eta_{th} q}{a} = \frac{\eta_{th} L}{a V^2} \quad (89)$$

plays a preponderante part. In particular, the propulsive efficiency η_p depends only on Q by the formula

$$\eta_p = \frac{1}{Q} \left[\sqrt{1 + 2Q} - 1 \right] \quad (90)$$

This shows that η_p increases from zero to 1, when Q decreases from infinity to zero, that is, when V or a (η_{th} being positive and finite) increases indefinitely. In the same conditions, the overall efficiency η_g increases from zero to the limit η_{th} below unity. Efficiency η_g and η_p are therefore always less than unity, contrary to what had been established earlier in the case of the explosive rocket. This arises from the fact that, owing to the preceding approximation which disregards the unity beside number a , the mass of fuel in the corresponding mass of burnt gases is neglected. Efficiencies η_g and η_p are then comparable to η_e and η_r which have been rationally defined in article 14.

25. Propulsive efficiency

According to (90), this efficiency depends only on parameter Q . Figure 17 shows this parameter Q plotted against a , V , and η_{th} for a type of fuel whose heat value (low and at constant pressure) is 11,000 cal/kg or $L = 45,900,000$.

It will be noted that for

η_{th} ranging between 0.30 and 0.70

a ranging between 20 and 150

V ranging between 25 and 200 m/sec (90 and 720 km/h) the parameter Q ranges between the extreme values $45,900,000 \times \frac{0.30}{150 \times 200^2} = 2.3$ and $45,900,000 \times \frac{0.7}{20 \times 25^2} = 2570$ for the fuel in question. The values of η_p for these extreme values are 0.592 and 0.0275, respectively. The variation of η_p as function of Q is given in table XI and plotted in figure 18.

Table XI

$Q =$	0	1	2	5	10	100	1000	∞
$\eta_p =$	1	0.732	0.62	0.464	0.358	0.132	0.045	0

Thus, it is seen that the propulsive efficiency η_p for a given thermal efficiency η_{th} cannot attain values comparable to the normal

efficiency of propellers (which is of the order of 0.60 to 0.80) unless the speed V is very high or else the corresponding consumption of air a is very great.

This is another instance of one of the essential characteristics of jet propulsion, already pointed out in article 14, namely, that the propulsion reaches high values only when the jet expels a large volume at the lowest possible speed of ejection with respect to the speed of propulsion, which is obtainable by increasing the latter. When η_p alone is considered, an increase in this efficiency involves the reduction of the thermal efficiency η_{th} , since it also reduces the parameter Q . This is entirely natural, after the foregoing, because the speed of ejection of the burnt gases is lowered. But, it also lowers the over-all efficiency, as will be shown later.

26. Over-all efficiency η_g

According to formula (88), the over-all efficiency η_g for a given fuel depends upon the thermal efficiency η_{th} and the ratio $q/a = L/aV^2$, that is, on the product aV^2 . It is readily apparent that η_g increases when η_{th} increases, when the dilution a increases, when V increases.

Table XII gives the over-all efficiency η_g as function of V and η_{th} on the basis of the consumption of a fuel whose heat value is 11,000 cal/kg and a dilution a either restricted to a minimum $a = 15$ or 5 times greater than this low theoretical limit (that is $a = 75$).

Table XII
Over-all Efficiency η_g

	$a = 15$			$a = 75$		
	$\eta_{th} = 0.2$	0.4	0.6	$\eta_{th} = 0.2$	0.4	0.6
m/sec						
$V = 50$	0.017	0.025	0.031	0.037	0.053	0.066
100	.033	.048	.0595	.066	.099	.1245
150	.0475	.070	.087	.090	.138	.1765
200	.060	.090	.113	.1095	.172	.222
250	.0725	.109	.1375	.1245	.202	.263

These data have been plotted in figure 19, along with the zone of the normal values of the over-all efficiency η_g of the conventional engine-propeller system, ordinarily ranging between 0.17 and 0.225.

This diagram brings out the unfitness of the rocket as substitute for the engine-propeller system at the speeds now reached in aviation. On the other hand, the superiority of the rocket is manifest at the high speeds of the order of 700 to 1,000 km/h if the rocket in these conditions is assured of a sufficiently high thermal efficiency η_{th} with a fairly great dilution a .

27. Thermal efficiency of the true rocket

The combustion of the fuel by means of air must be effected by compression of the combustible mixture as in a heat engine. The thermal efficiency is largely dependent upon the rate of this preliminary compression.

Different types of generators may be conceived. In particular, the compressor of the generator may be actuated by a standard engine (with carburation or injection) whose exhaust may be added to that of the rocket, but without appreciable effect with respect to the propulsive reaction; or a portion of the expansion of the gases intended for the rocket may be utilized to drive an engine (reciprocating or rotary) engaging the compressor. These two solutions are compared.

For the sake of simplicity, the study is restricted to the method of combustion at constant pressure (theoretically realized in the Diesel engine) because it supplies the most advantageous thermodynamic cycle at a specified maximum pressure in the heat machine. In addition to that, only one fuel is considered, that is, the fuel studied exhaustively by Rey and which is similar to the illuminating oil called kerosene in the United States, and for which a number of calculations had been made and the data of which are used in this report.

Before any evaluation of the possible thermal efficiency it is readily apparent that, if a dilution a of the order of those realized in gasoline or heavy oil engines and consequently, fairly low (say, a of the order of 20 to 35) is involved, it should be possible to reach a thermal efficiency for the rocket superior to that of a corresponding reciprocal engine because the principal part of the expansion is effected in the nozzle of the rocket (where the energy losses are, or can be, nearly zero) and this expansion is pushed to atmospheric pressure.

It is justified to anticipate that, with a weak solution of, say, 20 to 35, it should be possible to obtain a thermal efficiency η_{th} of at least the order of 0.35 to 0.40 with a true liquid-fuel rocket.

Adopting the figure 0.40 for a dilution $a = 30$, formula (88) shows that the corresponding rocket can attain an over-all efficiency equal to that of the best engine-propeller system used at present in aviation (engine consuming 210 g of fuel per hp/h, propeller efficiency 0.75), that is, $\eta_g = 0.28 \times 0.75 = 0.21$, for a speed $V = 422 \text{ m/sec} = 1520 \text{ km/h}$, a speed at which the present-day propellers will undoubtedly have but an insignificant efficiency.

28. Theoretical cycle

Rey's kerosene (Bulletin de l'Association technique maritime et aeronautique, Paris, 1928) which is used as typical fuel is defined by the following composition by weight:

40 percent of nonane (C_9H_{20})

25 percent of decane ($C_{10}H_{22}$)

15 percent of tridecane ($C_{13}H_{28}$)

20 percent of hexadecane ($C_{16}H_{34}$)

Its low heat value at constant pressure in standard conditions ($p_a = 1.033 \text{ kg/cm}^2$, $T_a = 273 + 15 = 288^\circ$) is $L = 11,500 \text{ cal/kg}$, or in M.K.S. units, $L = 425 \times 9.81 \times 11,500 = 48,000,000 \text{ kg per unit of mass (mass of 9.81 kg)}$.

Its complete combustion requires a minimum weight of air (23.6 percent O, 76.4 percent N) equal to 14.68 kg per 1 kg of fuel.

The molecular specific heats for gases or vapors are those given by Kast, according to the experiments by Pier and Bjerrum, that is, (in cal/mol-kg)

for O_2 , N_2 , and air: $C = 6.535 + 0.0009 T$

for H_2O (vapor): $C = 4.815 + 0.0043 T$

for CO_2 : $C = 10.665 + 0.00116 T$

The molecular or Avogadro constant for perfect gases is taken equal to

$$R = C - c = 1.985 \text{ (cal/mol-kg)}$$

On this basis, the characteristics of the theoretical cycle of a mixture of 1 kg of the preceding fuel and a specific amount of air with more or less excess can be computed; the said cycle comprising

(1) A reversible adiabatic compression of the air and a corresponding compression of the kerosene in the liquid state, the latter being negligible as to energy input and temperature rise experienced by the fuel

(2) An adiabatic and complete combustion at constant pressure

(3) An adiabatic and reversible expansion of the products of the preceding combustion, pushed to atmospheric pressure

In the following, the results of the calculation of this theoretical cycle, based upon the above data with strict consideration of the variation of temperature with the specific heat are indicated. The care given to the execution of these calculations should not cause any illusion regarding the significance of the results. It simply aims at freeing the latter from all causes of error other than that, already perceptible but which remains unknown, which include the values adopted for the specific heats, which result in the latest determinations. Table XIII gives the data in question. It shows, for different compression ratios $\lambda_c = p_c/p_a$ and for different values $\alpha = \frac{a - a_0}{a}$ of the corresponding excess of air in the combustible mixture

The temperature T_b of the air at the end of compression

The temperature T_c of the mixture at the end of the combustion

The temperature T_d of the mixture at the end of the expansion

The energy C_a (in cal) absorbed by the compression of the air, that absorbed by the compression of the kerosene being regarded as negligible

The energy C_d (in cal) produced by the expansion of the burnt gases

The effective energy C_e (in cal) of the cycle, obtained by the difference $C_e = C_d - C_a$

Lastly, the thermal efficiency $\eta_{th} = \underline{C}/L$ of the theoretical cycle involved (L evaluated in cal, or $L = 11,500$)

These data refer to the initial conditions

$$p_a = 1 \text{ kg/cm}^2 \quad T_a = 273 + 15^\circ = 288^\circ$$

Table XIII

Theoretical Cycle at Constant Pressure

α	$\lambda_c = p_c/p_a$	T_b	T_c	T_d	\underline{C}_a (cal)	\underline{C}_d (cal)	\underline{C}_e (cal)	η_{th}
0	15	621	2,860	1,669	1,239	6,555	5,316	0.4615
	30	751	2,925	1,472	1,650	7,925	6,275	.545
	45	841	2,973	1,372	1,981	8,711	6,730	.585
1	15	621	1,947	1,026	2,478	8,278	5,800	.503
	30	751	2,032	903	3,300	10,012	6,712	.584
	45	841	2,103	848	3,962	11,155	7,193	.625
2	15	621	1,561	783	3,717	9,607	5,890	.512
	30	751	1,645	690	4,950	11,835	6,885	.598
	45	841	1,730	654	5,943	13,368	7,425	.645
3	15	621	1,353	667	4,956	10,900	5,944	.517
	30	751	1,450	594	6,600	13,628	7,028	.611
	45	841	1,545	572	7,924	15,562	7,638	.664

The variation in thermal efficiency η_{th} is represented by the curves of figure 20. These curves confirm the well-known increase in η_{th} with the compression ratio λ_c . They also show that η_{th} increases, for a fixed λ_c , when the corresponding excess of air a in the combustible mixture is increased. This result, obtained in spite of the general reduction of the temperatures of the cycle, is not at variance with the laws of thermodynamics. It is simply an unjustified comparison of internal-combustion engines with properly cyclic machines as it is sometimes pretended that the thermal efficiency of the former is always so much higher as the temperatures of the cycle are, in general, so much higher. On assuming that the excess of air increases indefinitely, these temperatures drop more and more and it becomes increasingly justified to admit the same heat properties for the burnt gases as for pure air and, at the same time, to consider the specific heats of the former and latter, regarded as identical, as constant. A

classical reasoning makes it possible to establish that the thermal efficiency of the theoretical cycle tends then toward the value

$$\eta_{th} = 1 - \lambda_c^{-\frac{\gamma-1}{\gamma}} \quad (91)$$

with γ of the order of 1.35 to 1.4.

With this formula, the following values of η_{th} are obtained:

	$\lambda_c = 15$	30	45
$\eta_{th} = \begin{cases} \text{for } \gamma = 1.35 \\ \text{for } \gamma = 1.40 \end{cases}$	$\begin{matrix} 0.505 \\ .540 \end{matrix}$	$\begin{matrix} 0.587 \\ .622 \end{matrix}$	$\begin{matrix} 0.628 \\ .663 \end{matrix}$

The variation of η_{th} obtained by a very simple reasoning is in very good agreement with that given in table XIII and the curves of figure 20, which were obtained from calculations as precise and exact as possible.

In any case, the efficiencies calculated above concern the theoretical cycle chosen as model and it remains to be seen how this cycle can be produced in practice and which then becomes its thermal efficiency.

29. Realization of the theoretical cycle

The different methods conceivable can be grouped in two classes:

(a) Integral cycle, in which all elements of the air-fuel mixture go through the same cycle. - In this case, the products of combustion must transfer part of their effective expansion energy, necessary to drive the compressor, to a driving mechanism. This is the complement of the energy of the total expansion which is expressed by the increase in kinetic energy of the active bodies at the rocket outlet, whence the propulsive reaction of the latter results. In this integral cycle, the engine and the compressor may, themselves, be of the reciprocating, the piston, or rotatory type (turbomachines).

(b) Divided cycle. - In this case, the engine driving the compressor of the rocket consumes a combustible mixture which is subjected in the said engine to a more or less different cycle from that which the mixture, intended for the rocket itself, undergoes. In this case, the engine can operate with combustion at constant pressure or constant volume, whether it is of the reciprocating or of the rotary type.

Its exhaust can be made direct, without appreciable propulsive effect as for the usual airplane engines, or else be combined with that of the rocket, the corresponding two jets forming a more or less homogeneous mixture. No attempt is made to study the various combinations in detail; only the elementary system corresponding to the engine with exhaust independent and without appreciable propulsive effect is analyzed.

30. Thermal efficiency of the rocket realizing the integral cycle

The subsequent study is concerned with the case where the engine is of the piston type and that where it is of the turbine type.

(A) Case of piston-type engine.- In this case it is logical to perform the function of the compressor in the engine. In short, the engine then is like the classical internal-combustion engine of the Diesel type. It only differs from it by the fact that its expansion is cut short and its exhaust regulated in such a way that it can produce no effective energy at the outside; the motive energy of the expansion rigorously balances the resistant energies, the most important of which is that of the preliminary compression of the air.

Different designs may be visualized. If the exhaust is at atmospheric pressure (fig. 21), the expansion must be very short and the engine compressor necessarily with uneven strokes, if of the four-stroke cycle type. Thus it will seem preferable to resort to two-stroke cycle operation with scavenging in one direction, and scavenging air reservoir upstream. Besides, the exhaust is accompanied by impulses or pulsations as in the conventional engines and this fact gives the exhaust of the rocket a periodic behavior, while vitiating the efficiency of its nozzle. When, to avoid this draw-back, the exhaust of the engine compressor is effected at rigorously constant pressure, recourse must be had to an engine operating on the principle of the diagram of figure 22. The two-stroke cycle must then be abandoned in favor of a more complicated system of distribution.

It will be noted that it is also possible to effect a cycle such as that of figure 22 by means of two cylinders with transfusion from one to another, each being able to operate at two strokes and the whole forming the equivalent of a single four-stroke cylinder.

Whatever these variants may be, the study of which must be very carefully carried out if an engine of this type is to be realized, an attempt is made to define the thermal efficiency that may be obtained in a rocket with an engine operating according to the cycle illustrated in figure 22.

The visualized engine compressor, is, like any other heat engine, subject to losses due to clearance, to stratification, to the walls, to incomplete combustion etc. Its actual operation is compared with a fictitious operation involving

(a) An irreversible adiabatic compression of pure air with a compression efficiency ρ_a and which absorbs, in consequence, a real energy $C_a' = C_a/\rho_a$ (all calculations being referred to the consumption of 1 kg of fuel, that is, per unit of weight rather than mass in

the M.K.S. system (table XIII)). The temperature of the mixture at the end of compression, that is, at pressure $p_c = \lambda_c p_a$, is then raised to T_b' instead of T_b ($T_b' > T_b$);

(b) An irreversible and incomplete adiabatic compression at constant pressure, releasing but the fraction $(1 - l)$ of the heat value L . The corresponding loss l ($0 < l < 1$) totals, fictitiously, the effect of the losses due to the incomplete combustion and to the wall effect in the real engine.

(c) An irreversible adiabatic expansion accomplished with an expansion efficiency ρ_d' and furnishing an effective energy equal to the energy \underline{C}_a' absorbed by the compression. The final pressure $p_d' = \lambda_d p_a$ of this partial or primary expansion is, therefore, determined by the condition that the reversible adiabatic expansion from p_c to p_d' supply a theoretical energy equal to $\underline{C}_a' / \rho_d' = \underline{C}_a / \rho_a \rho_d'$. The actual temperature T_d' at the end of the expansion is then easily computed by writing that the fraction $(1 - \rho_d')$ of the energy of the theoretical expansion is equal to the temperature rise of the gases at constant pressure, to the temperature \bar{T}_d' (corresponding to the perfect expansion), and to the desired temperature T_d' (corresponding to the real expansion).

The conditions of the gases supplied by the engine to the nozzle of the rocket, following their primary expansion in the engine are (p_d', T_d') . Then they experience a secondary expansion from p_d' to p_a in the nozzle of the rocket, with an expansion efficiency ρ_d'' in respect to the reversible expansion, which supplies an energy \underline{C}_d'' . The real expansion produces, therefore, the energy $\underline{C}_d'' = \rho_d'' \times \bar{\underline{C}}_d''$ and raises the expanded gases to the final rocket exhaust temperature T_e .

The thermal efficiency η_{th}' of the true rocket to be evaluated is, by definition, equal to \underline{C}_d'' / L when the air is trapped upstream in the state (p_a, T_a) without appreciable kinetic energy.

The calculation was carried out by this method for two widely varying values of excess air α in the mixture, namely:

$\alpha = 0$ or exact mixture (limiting case)

$\alpha = 3$ or mixture much diluted, which undoubtedly is of little advantage because both the bulk and the weight per horsepower of the system become prohibitive.

In this calculation, the fairly high value of $\rho_a = \rho_d = 0.90$ was uniformly adopted for the efficiency of compression and expansion in the reciprocating engine, which takes account of the good efficiency of

the piston engine for the high pressures at compression or adiabatic expansion. For the fictitious loss l , the high value $l = 0.20$ was chosen for the exact mixture ($\alpha = 0$) because of the high temperatures of the cycle which call for intensive cooling of the walls and the pistons, and the low value $l = 0.10$ for the very diluted mixture ($\alpha = 3$) by reason of the much lower temperatures of the cycle. For the efficiency of expansion in the rocket nozzle, the value $\rho_d'' = 0.95$ was adopted. Lastly, the outside temperature is assumed to be standard, that is, $T_a = 273 + 15 = 288^\circ$.

The results of the calculation are given in table XIV.

Table XIV

		T_b' (deg)	T_c' (deg)	C_a' (cal)	\bar{T}_d' (deg)	T_d' (deg)	$\lambda_d' = P_d'/P_a$	\bar{C}_d'' (cal)	C_d'' (cal)	T_e (deg)	$\eta_{th}' = C_d''/L$
Strict mixture $\alpha = 0$ $l = 0.20$	$\lambda_c = 15$	661	2,489	1,377	2,209	2,237	8.16	4,150	3,940	1,476	0.342
	30	803	2,566	1,835	2,196	2,233	13.45	4,857	4,705	1,331	.409
	45	903	2,622	2,202	2,189	2,234	17.80	5,251	4,987	1,256	.433
Mixture very diluted $\alpha = 3$ $l = 0.10$	$\lambda_c = 15$	661	1,314	5,508	931	971	3.85	4,750	4,508	679	.392
	30	803	1,427	7,340	922	974	5.35	5,385	5,110	643	.444
	45	903	1,514	8,808	911	973	6.015	5,732	5,445	621	.473

The variation of η_{th}' as function of λ_c is represented by the curves in figure 23, along with the efficiency η_{th} of the corresponding theoretical cycle for comparison. The diagram shows the marked drop in efficiency of the actual compared to the theoretical system serving as model. This drop averages about 25 percent for the chosen examples and assumptions.

The conclusion is that, with an excess of air ranging between 0 and 300 percent (or λ between 15 and 60) and a compression ratio λ_c between 15 and 45, a system of the reciprocating type in question should be able to develop a thermal efficiency η_{th}' of the order of 0.40, probably ranging between 0.37 and 0.43.

Up to now the energy of the compression had been considered as being effected by starting from an initial state where the air to be compressed is largely devoid of speed with respect to the engine, that is, by assuming the speed V to be relatively low. If the speed V is very substantial, the air trapped by the compressor is, once its speed is gone, in a state where its compression is increased and where, in consequence, a part of the total compression λ_c is already realized. The energy required from the engine is thus diminished and the kinetic energy developed by the gases in the expansion nozzle is raised, but it is easy to see that this increase corresponds to the initial kinetic energy of the air which is slowed down before the compressor, the efficiency of the various compressions and expansion being, however, taken into account. Upstream from the compressor is the air inlet which captures the free air and operates as diffuser, with the comparatively mediocre efficiency of compression characteristic of diffusers, in general. The efficiency η_{th}' indicated in table XIV is then subject to a certain reduction.

This remark implies that, when V becomes appreciable, (higher than 150 - 200 m/sec for example), the reciprocating compressor of the system considered above loses part of the advantages which justify its fundamental aptitude for obtaining high compression with reduced losses.

(B) The case where the engine is a turbomachine.- In this case, it is logical to combine the turbine with a compressor that is also rotatory, that is, a turbocompressor.

Visualize such a system which represents a turbine with balanced internal combustion, hence, produces no effective energy on its shaft. The obligation to operate with fairly low temperatures on the buckets of the compressor and of the turbine results in limiting the compression ratio λ_c and utilizing a considerable excess of air. In addition, the first rotor of the turbine must be installed, during the expansion of the burnt gases, at a sufficiently low pressure.

On the assumption that speed V is not too high so that the kinetic energy of the air upstream from the compressor can be neglected with respect to the theoretical energy \bar{C}_a of the compression to which it is subjected, the compressor of efficiency ρ_a (on the connecting shaft with the motive turbine) absorbs an energy $\bar{C}_a' = \bar{C}_a/\rho_a$, the energy that should be furnished to it by the motive turbine.

Let \bar{C}_d represent the (theoretical) energy of the perfect and total expansion of the burnt gases after combustion at degree $(1 - l)$, that is, from the conditions (p_c, T_c') . Let ρ_d' denote the specific efficiency of the turbine. The latter supplies a real energy $\bar{C}_d' = \bar{C}_d \rho_d'$ and takes, at complete expansion, a part $\bar{C}_d' = \bar{C}_d \rho_d'$ of the energy \bar{C}_e of this ideal expansion. The complement $\bar{C}_d'' = \bar{C}_d - \bar{C}_d'$ (the temperature rise of the gases at the outlet of the real turbine with respect to the ideal expansion being disregarded) is available for the nozzle of the rocket and the latter transforms it, with a proper efficiency ρ_d'' into kinetic exhaust energy $\bar{C}_d'' = \rho_d'' \bar{C}_d''$. Of course, in the foregoing the compressor, the turbine, and nozzle of the rocket are assumed to be actually adiabatic. In fact, the heat losses of such devices by conduction and radiation should be very small in the system at the low temperatures visualized and, therefore, can be disregarded. The real thermal efficiency to be computed is then

$$\eta_{th}' = \frac{\bar{C}_d'}{L} = \frac{\rho_d''}{L} \left[\bar{C}_d - \frac{\bar{C}_a}{\rho_a \rho_d'} \right] \quad (92)$$

When ρ_a, λ_c , and α are fixed, \bar{C}_a and \bar{C}_d are defined and the preceding formula shows that η_{th}' is so much higher as the turbine efficiency ρ_d' itself is higher. However, it is expedient to note that, in the corresponding energy loss $(1 - \rho_d')$ through the turbine, a fraction k_v ($0 < k_v < 1$) corresponds to the loss by the remaining kinetic energy while the complementary fraction $(1 - k_v)$ corresponds to the other losses (internal and external) of the turbine.

For the rocket, the kinetic energy of the gases at the turbine exit, that is, the quantity $[k_v(1 - \rho_d')\bar{C}_d']$ is directly usable in the rocket nozzle and should be added to the energy \bar{C}_d'' .

On this basis, formula (92) is more accurately expressed by

$$\begin{aligned} \eta_{th}' &= \frac{\rho_d''}{L} \left[\bar{C}_d'' + k_v(1 - \rho_d')\bar{C}_d' \right] \\ &= \frac{\rho_d''}{L} \left\{ \bar{C}_d - \left[1 - k_v(1 - \rho_d') \right] \frac{\bar{C}_a}{\rho_a \rho_d'} \right\} \end{aligned} \quad (93)$$

Accordingly, with ρ_a , λ_c , and α fixed, it is seen that the maximum of η_{th}' corresponds to the maximum of the quantity

$$r = \frac{\rho_d'}{1 - k_v(1 - \rho_d')} \quad (94)$$

For a turbine of the chosen type, ρ_d' and k_v , and, consequently r , depend upon the head h (that is, the pressure level of the turbine with respect to the pressure at start p_c) and upon the turbine speed (that is, in ratio ω of its peripheral speed to the theoretical speed of the fluid due to the head h).

Within the range of the variables h and ω , the quantity r has a certain maximum which does not necessarily correspond to the same conditions (h_0 , ω_0) as the maximum of ρ_d' considered alone. Theoretically, this is due to the effect of external losses, because, if only the internal losses are considered, the maximum of r corresponds almost rigorously to the minimum of the loss by the remaining speed. These are the conditions that should prevail when, however, the optimum speed ratio ω_0 of the turbine thus defined is practically realizable.

For the approximate evaluation of η_{th}' under consideration, it is simply stated that quantity r is given a value slightly higher than that of the efficiency ρ_d' of a good turbine.

Reverting to the example of the theoretical cycle corresponding to table XIV, in order to obtain the fairly low temperatures which the machine requires, consider the case $\alpha = 3$, that is, a dilution of the mixture that can be exceeded only at the expense of increased, perhaps excessive, bulk and weight of the system.

Supposing the total combustion is ($l = 0$), as it can be realized in a system of this type, and the values of λ_c are limited to between 10 and 15 which actually are at the limit of the range of application of turbocompressors. Such devices are ordinarily designed for quasi-isothermal operation and, as such, compared to the ideal isothermal compressor by means of an efficiency ρ_i , completely different from ρ_a . In a study of the internal combustion turbine cited elsewhere, it had been shown that it was advisable to adopt the principle of adiabatic compression and to design the compressors in consequence. The data available at the present time do not permit an evaluation of the efficiency ρ_a (with respect to the ideal adiabatic) that may be hoped for from such compressors. Still, it seems safe to concede that a value of the order of 0.75 for a compression ratio of the order of 10 could be obtained. The consistent progress with turbomachines which helps to speed up the arrival of the internal combustion turbine will undoubtedly bring efficiencies of the order of 0.80 to 0.85 in the very near future.

To maintain the margin of possible progress, ρ_a is given the hypothetical values 0.7, 0.8, 0.9. From this follow for $\lambda_c = 10$ and $\lambda_c = 15$ the characteristics given in table XV.

Table XV

Real Cycle at Constant Pressure ($\alpha = 3$, $T_a = 288^\circ$)

λ_c	ρ_a	T_b' (deg)	T_c' (deg)	$\underline{C}_a' = \underline{C}_a/\rho_a$ (cal)	\underline{C}_d (cal)
10	0.9	577	1,298	4,060	9,298
	.8	611	1,328	4,568	9,522
	.7	657	1,368	5,220	9,802
15	.9	661	1,372	5,291	11,027
	.8	707	1,411	5,952	11,362
	.7	766	1,462	6,803	11,807

So, quantity r characterizing the utilization of the turbine is given the values 0.75 and 0.85, which apparently ought to include the cases practically obtainable without abnormal difficulties. Formula (94) then can be used to compute the desired η_{th}' . Table XVI gives the data of this calculation which was made on the assumption of the expansion nozzle efficiency equal to $\rho_d'' = 0.95$.

Table XVI

Thermal Efficiency of the True Turborocket

($\alpha = 3$, $T_a = 288^\circ$, $\rho_d'' = 0.95$)

λ_c	ρ_a	r	η_{th}'
10	0.9	0.85	0.372
		.75	.3195
		.85	.3415
	.8	.75	.2825
		.85	.3015
		.75	.234
15	.9	.85	.396
		.75	.3275
		.85	.3595
	.8	.75	.2825
		.85	.314
		.75	.2255

These data are represented in the diagram of figure 24.

It is evident from this diagram that:

- (1) Within the range of ρ_a , ρ_d' , and λ_c in question, η_{th}' ranges between 0.22 and 0.40.
- (2) At equality of the apparent turbine efficiency r (between 0.85 and 0.75 by assumption), especially if the latter differs little from 0.75 and from the compressor efficiency ρ_a , there is little reason to push the compression λ_c beyond 10, especially as such an increase would undoubtedly lower the anticipated compressor efficiency ρ_a .
- (3) The value of r , for fixed λ_c and ρ_a has a marked effect on the thermal efficiency η_{th}' of the complete system. This apparent thermal efficiency of the turbine should therefore have the highest possible value.

To illustrate: take a case that is easily realizable, that is, defined by $\lambda_c = 10$; $\rho_a = 0.75$. It is seen that η_{th}' will range between 0.259 and 0.322 when r itself ranges between 0.75 and 0.85.

Consider, for example, a specific turbine efficiency equal to 0.75 and see whether it is practically feasible and what value r reaches. In this instance, $\underline{C}_a' = 4878$ cal; $T_b = 633^\circ$; $T_c' = 1347^\circ$; $\underline{C}_d = 9645$ cal.

In order to reduce the temperature at the turbine buckets to a minimum, it is expedient to use a single acting, single rotor turbine.

The specific turbine efficiency ρ_d' being assumed equal to 0.75, this turbine should utilize the part of the expansion corresponding to a theoretical energy for perfect expansion, equal to $\underline{C}_a'/\rho_d' = 6505$ cal. This preliminary expansion ranges between the upstream pressure $p_c = 10p_a$ and the intermediary pressure $p_d' = 2.71p_a$, which is that which must prevail in the junction between the distributor and the turbine wheel.

If this primary expansion is by reversible adiabatic, the final (absolute) temperature of the burnt gases is 945° (or 672° C). In fact, the temperature of the gases back of the buckets is slightly higher by reason of the irreversibility of the true expansion.

The theoretical speed due to the head feeding the turbine is $W = 953$ m/sec. With i denoting the slope of the buckets of the distributor, μ and λ the throttling coefficients in the distributor and in the wheel, and U the peripheral speed of the rotor, the internal

efficiency ρ_1 (which disregards the external losses of the turbine) of the single rotor, single action turbine is given by the formula

$$\rho_1 = 2(1 + \lambda) \frac{U}{W} \left(\mu \cos i - \frac{U}{W} \right) \quad (95)$$

The values for a good impulse turbine operating at total injection are

$$i = 15^\circ \quad \mu = 0.98 \quad \lambda = 0.87$$

Conceding that the outside losses are reduced to 7 percent of the theoretical horsepower of the turbine, it is necessary, in order to obtain the specific efficiency ρ_d' admitted, a priori, equal to 0.75, that the internal efficiency ρ_1 reach 0.82. Formula (95) shows that, in order to obtain this result, the ratio U/W must have a value of 0.403 and that the peripheral speed U of the rotor must be $0.403 \times 953 = 384$ m/sec.

A quick calculation indicates that the quantity k_v in the expression (94) of the coefficient r , for the impulse turbine, is given by the formula

$$k_v(1 - \rho_d') = (1 + \lambda)^2 \left(\frac{U}{W} \right)^2 + \lambda^2 \mu^2 - 2\lambda \mu \cos i (1 + \lambda) \frac{U}{W} \quad (96)$$

For the example in question, this formula gives the value $k_v(1 - \rho_d') = 0.048$ and formula (94) gives $r = 0.789$.

Lastly, for the whole system, characterized by $\lambda_c = 10$; $\alpha = 3$; $\rho_a = 0.75$; $\rho_d' = 0.75$; $r = 0.789$; $\rho_d'' = 0.95$ the thermal efficiency η_{th}' reaches the value 0.285, with due allowance for the partial recovery of the kinetic energy left in the turbine outlet in the nozzle of the rocket.

This efficiency is of the order of that of the best aircraft engines of today and is not obtained, except with a system operating at relatively low pressures and temperatures with respect to those of the usual internal combustion engines.

It would be improved considerably if the ρ_a and ρ_d' of the turbomachines employed could be made superior to the efficiencies admitted in the foregoing example. In a similar case, it is found that it is advisable to force the dilution ($\alpha > 3$) of the combustible mixture and the compression ($\lambda_c > 10$) which precedes the combustion, simultaneously. It is true that the above gas turbine raises an appreciable difficulty in the sense that its peripheral speed must attain 384 m/sec

at a gas temperature in contact with the buckets of the order of 672°C . But it is proper to note that the severity of these conditions, comparable to those in which some exhaust gas turbines used as superchargers actually operate, is considerably ameliorated by either forcing the excess air of 300 percent ($\alpha = 3$) admitted in the previous example, or else contemplating the operation of the system at sufficiently high altitude because the decrease in T_a then reduces all temperatures of the cycle in a roughly proportional manner.

Note. - All of the above calculations assume, as for the reciprocal engine considered elsewhere, the compression of the initial kinetic energy of the air to be slight or quasi-negligible at entry in the compressor compared to the energy required from the compressor, as is the case when the speed V is below 150 to 200 m/sec, for example.

In the case of the turborocket with very high speed, the obtainable thermal efficiency η_{th}' is increased, all other things (α , λ_c , ρ_a , ρ_d' , and ρ_d'') being equal.

31. Thermal efficiency of the rocket by divided cycle

In this system, the air compressor that supplies the combustion chamber of the rocket is driven by an engine consuming, for its own operation and in independent manner, a certain portion of the fuel. It is assumed that this engine does not contribute by its exhaust to the propulsive reaction developed by the exhaust of the rocket itself. Individually, engine and compressor may be of any reciprocating or rotary type.

The fuel consumed by the rocket is, as before, per 1 kg.

C_a' = the real energy absorbed by the compressor, of the efficiency ρ_a

C_d = the theoretical energy of expansion of the combustion gases in the rocket

C_d' = the corresponding real energy converted to kinetic energy in the nozzle, the expansion coefficient of which is ρ_d

With η_m denoting the effective thermal efficiency of the engine (at the compressor drive shaft), the latter should produce an energy equal to C_a and consume, to this effect, the amount of fuel $\frac{C_a'}{\eta_m L}$, C_a' and

L being expressed in cal/kg of fuel utilized, which were assumed to be identical for the rocket and the auxiliary engine.

The useful effect of the whole is represented by the kinetic energy developed by the gases in the nozzle of the rocket, or $\underline{C}_d' = \rho_d \underline{C}_d$. To obtain the thermal efficiency of the whole, it should be referred to the heat value of the total amount of fuel consumed $(1 + \underline{C}_a' / \eta_m L)$.

The desired efficiency is then given by the simple formula

$$\eta_{th}' = \frac{\eta_m \rho_d \underline{C}_d}{\eta_m L + \underline{C}_a'} = \frac{\eta_m \rho_a \rho_d \underline{C}_d}{\underline{C}_a + \eta_m \rho_a L} \quad (97)$$

For the engine, an efficiency $\eta_m = 0.27$ may be assumed; this corresponds to engines consuming around 220 gallons of gasoline per hp/hr.

For the efficiency ρ_d of the expansion nozzle, the value of $\rho_d = 0.95$ is assumed.

The provision for the rocket itself still is the cycle with isobaric combustion of the fuel mixture analyzed previously while assuming the several parts of the rocket (compressor, combustion chamber, nozzle) practically adiabatic. Here, also, it is advisable to limit the temperatures attained in the compressor and in the combustion chamber.

It is further assumed that $\alpha = 3$, and for this 300-percent excess of air in the mixture consumed by the rocket, a compression ratio λ_c ranging between 10 and 15 is visualized. Lastly, the combustion is regarded as being practically complete, while the surrounding temperature is $T_a = 273 + 15 = 288^\circ$.

For the compressor efficiency ρ_a (with respect to the adiabatic), values ranging between 0.7 and 0.9 are considered, since this range appears to include the efficiencies which good compressors with pistons or wheels can obtain.

On these premises, the obtainable η_{th}' reaches, according to the values assumed for λ_c and ρ_a , the values given in table XVII and plotted in figure 25.

Table XVII

Rocket with Divided Cycle

$$(\alpha = 3; T_a = 288^{\circ}; \eta_m = 0.27; \rho_d = 0.95; \\ L = 11,500 \text{ cal/kg})$$

λ_c	ρ_a	T_b' (deg)	T_c' (deg)	C_a (cal)	C_a' (cal)	C_d (cal)	η_{th}'
10	0.9	577	1,298	3,652	4,060	9,298	0,332
	.8	611	1,328		4,568	9,522	,3175
	.7	657	1,368		5,220	9,802	,302
15	.9	661	1,372	4,7555	5,291	11,027	,336
	.8	707	5,952		5,952	11,362	,321
	.7	766	6,803		6,803	11,807	,305

From figure 25, it is evident that

(1) Within the stipulated range of ρ_a , ρ_d , and λ_c , the efficiency η_{th}' lies between 0.30 and 0.34

(2) At equal values of ρ_a , it serves no useful purpose to raise the compression λ_c above 10 especially as such an increase would undoubtedly lower the efficiency ρ_a to be attained by the compressor

It is essential to note that, in this instance, a thermal efficiency superior to that of the best aircraft engine is obtained. And this result is achieved here without encountering any difficulty as regards the realization of the expansion of the burnt gases, which is effected entirely in a nozzle without interposition of paddle wheels.

As before, it is well to remember the assumption that the kinetic energy of the air before entering the compressor was almost negligible compared to the energy which the compressor must supply. Therefore, the computed efficiencies apply only to rockets at speeds below 150 - 200 m/sec. For high-speed rockets, the initial kinetic energy of the air which, moreover, permits obtaining a slightly improved thermal efficiency η_{th}' , must be taken into account.

32. Comparison of rockets with integral cycle and rockets with divided cycle

From the foregoing study the following conclusions can be drawn:

(1) The obtainable thermal efficiency η_{th}' appears to range between the following limits

Rocket with integral cycle	Engine-piston compressor	minimum dilution ($\alpha = 0$)	$0.34 < \eta_{th}' < 0.43$
	Turborocket (expansion turbine and turbocompressor)	maximum dilution ($\alpha = 3$)	$0.39 < \eta_{th}' < 0.47$
Rocket with divided cycle	Separate reciprocating engine- vane or piston type compressor	normal dilution ($\alpha = 3$)	$0.22 < \eta_{th}' < 0.40$
		normal dilution ($\alpha = 3$)	$0.30 < \eta_{th}' < 0.33$

(2) The engine-piston compressor system producing the integral cycle appears likely to develop the highest thermal efficiency. The latter increases constantly with increased dilution of the combustible mixture, but its drawback of size and weight appears to favor turbomachines.

(3) With greatly diluted combustible mixture, the turborocket surpasses the rocket with integral cycle, when the efficiency of the expansion turbine and of the turbocompressor which form it, is high enough. The realization of the first necessitates, it is true, the development of turbines operating at speeds and temperatures which are still a little above those of current practice, but the difficulties involved cannot be regarded as prohibitive.

(4) The rocket with divided cycle and greatly diluted mixture presents, in compensation for its correspondingly low efficiency, estimated at about 0.30 to 0.33, the particular advantage of being attainable with the least difficulty by using a standard reciprocating engine, a suitable compressor (compression of the order of 10), and combustion chambers and nozzles which, operating at relatively moderate temperatures, are comparatively easy to obtain.

33. Study of two typical examples of true liquid-fuel rockets

Case 1.- A so-called rocket A, with engine and piston compressor, and a dilution of combustible mixture limited to the value $\alpha = 0.5$ (50 percent excess of air) so as to keep size and weight from becoming excessive. On the other hand, in this type of system a fairly high compression of $\lambda_c = 40$ to 45 is admitted in order to improve the

efficiency as much as possible and to remain within the neighborhood of the operating conditions of Diesel engines. The efficiency η_{th}' may be assumed equal to 0.44.

Case 2.- A so-called turborocket B, for which is chosen the case of relatively easy realization, already discussed (in 30, B) and for which

$$\alpha = 3 \quad \lambda_c = 10 \quad \rho_a = 0.75 \quad \rho_d' = 0.75 \quad \rho_d'' = 0.95$$

For this rocket with low pressures and temperatures, the probable thermal efficiency η_{th}' has been estimated equal to 0.285. To insure greater accuracy, it is advisable to take the effect of the kinetic energy of the air before entering the compressor of the rockets into consideration. This initial energy is transformed in the orifice of the compressor operating as diffuser, into irreversible and adiabatic compression energy of the air, the velocity of which is damped. The diffuser efficiency with respect to the reversible adiabatic is generally mediocre and this efficiency is optimistically put at 0.75.

If a constant total compression ratio λ_c is maintained, the damped kinetic energy before the compressor lowers the compression energy which the latter must supply. The energy to be assumed at the last expansion of the burnt gases to operate the compressor is equally reduced and the complement of the expansion furnishes an accrued final kinetic energy at the rocket exit. The difference between the latter and the initial kinetic energy referred to the heat value is a measure of the thermal efficiency of the system.

The result, as is easily seen, is that, in the case of rocket A and when considering the values assumed for the different efficiencies of compression and expansion, the thermal efficiency decreases with the speed V , the total ratio of compression λ_c being assumed constant. The decrease is small. Besides, it can be avoided by increasing λ_c , for example, by maintaining a constant rate of volumetric compression in the engine compressor, which actually facilitates the design and use of the system.

It is further assumed that the thermal efficiency η_{th} of rocket A maintains the value 0.44 when the speed increases.

A first approximation¹⁰ of the variation of η_{th}' with speed V of rocket B is obtained by the following simple reasoning:

On the basis of the assumptions, the efficiency of compression in the diffuser which precedes the turbocompressor is found to have the same value (0.75) as the specific efficiency ρ_a of the compressor, the latter efficiency being assumed estimated with a negligible kinetic energy at induction (side of air intake operating as diffuser) as well as at discharge (side of feed of combustion chambers of the rocket). Consequently, whatever the contribution of effective energy absorbed by the diffuser-compressor unit may be, this total energy remains constant as long as the over-all ratio of the compression λ_c itself is constant. By the same argument, the state of the cyclic fluids at the end of compression undergoes no modification and the combustion takes place under constant conditions.

Thus, if W_a is the corresponding kinetic energy of the captured air, it is seen that, with respect to the case where it is zero or negligible, the motive energy required by the compressor from the turbine which drives it, is reduced to W_a . This turbine utilizes then but a small fraction of the theoretical expansion energy C_d , leaving the amount W_a/ρ_d' of this energy. The expansion nozzle utilizes this supplementary energy of the theoretical expansion with the expansion efficiency ρ_d'' .

Lastly, it is seen that the final kinetic energy increases to $\rho_d''W_a/\rho_d'$, when the initial kinetic energy increases to W_a . The thermal efficiency η_{th}' of the total thermodynamic cycle being, according to formula (4) and for the system in question, the ratio of the difference in the initial and final kinetic energy of the active bodies to the heat value L , it is seen immediately that the increase in η_{th}' with speed V is

$$\Delta\eta_{th}' = \frac{W_a}{L} \left[\frac{\rho_d''}{\rho_d'} - 1 \right] \quad (98)$$

¹⁰The approximation underestimates the variation of η_{th}' systematically, because first, it disregards the partial recovery, in the nozzle of the rocket, of the loss due to residual speed of the turbine situated upstream, and second, it likewise neglects the temperature rise of the gases at the turbine exit, which is due to the losses of the real cycle in the turbine. This rise affords a partial recovery of the said losses during a subsequent expansion and, notably, in the nozzle of the rocket. This effect, of secondary importance, has been neglected in all the calculations of the thermal efficiency of the cycles with fractional expansion.

This variation has the prefix of an increment, because the efficiency of the expansion turbine ($\rho_d' = 0.75$) is less than that of the ejection nozzle of the rocket ($\rho_d'' = 0.95$). Besides, this increment is quite small as indicated in the following tabulation of the η_{th}' values computed for rockets A and B at various speeds:

$V =$ (m/sec)	0	100	200	300	400	500
$\eta_{th}' =$	0.285	0.2866	0.2915	0.2997	0.3101	0.3258

These values are applied hereafter.

The over-all efficiency η_g of rocket A and B follows from the general formula (82), in which V is henceforth the only variable.

Thus, with Rey kerosene we get:

Rocket A: $a = 22.02$; $L = 11,500 \text{ cal/kg} = 48,000,000$
(M.K.S. system); $\eta_{th}' = 0.44$

Rocket B: $a = 58.72$; $L = 48,000,000$;
 $\eta_{th}' =$ values of the preceding table

With these data, formula (82) gives the η_g values indicated in table XVIII and plotted in figure 26.

Table XVIII

η_g Values for Rockets A and B

$V =$ (m/sec)	0	100	200	300	400	500
Rocket A: $\eta_g =$	0	0.0605	0.113	0.1575	0.197	0.231
Rocket B: $\eta_g =$	0	0.0735	0.129	0.1725	0.2085	0.240

The examination of figure 26 shows that

(1) The turborocket B has a better over-all efficiency than rocket A

(2) This superiority is, however, quite small, since it varies in value from 14 to 4 percent when V varies between 200 to 500 m/sec

(3) Rocket B, the most advantageous, does not begin to be comparable ($\eta_g = 0.17$) to the engine-propeller system until the speed of 1050 km/h is reached and then it certainly becomes superior to the best system of this type ($\eta_g = 0.225$) when V exceeds 1600 km/h

Such is the principal conclusion reached from the study of true rockets using an ordinary fuel such as kerosene. This conclusion is in accord with that obtained in the study of solid-fuel rockets. On the other hand, the kerosene rocket, at equal over-all efficiency, has a specific fuel consumption identical with that of an aircraft engine with gasoline or kerosene driving a propeller. This holds for the equivalence of heat values of the utilized fuels and gives a primordial advantage to the kerosene rocket over the explosive rocket. One of the most essential means of raising the over-all efficiency of the kerosene rocket consists, as for the explosive rocket, in raising its volume without unduly lowering its thermal efficiency.

Chapter II - Liquid-Fuel Rocket with Thrust Augmentation

34. Principle and operation

The liquid-fuel rocket with thrust augmentation differs from the corresponding plain rocket only by the addition of a thrust augments, as already indicated in the study of the explosive rocket with thrust augmentation.

The principle and the mode of operation is the same as for the explosive rocket. However, the thrust equations and the efficiencies are modified by reason of the corresponding air consumption a of the rocket proper.

35. Equations of operation of the rocket with thrust augmentation

With a' as the mass of air captured by the thrust augments per unit mass of fuel consumed by the rocket with thrust augmentation

$$\mu = a'/a \quad (99)$$

The ratio μ represents the air volume ratio (a') trapped by augments and (a) consumed by the rocket, the volume of fuel being negligible in comparison to the latter. To express the speed of

exhaust w_e , thrust T , intensity of thrust $t = T/S_e$, and finally the efficiencies η_p and η_g , of the rocket with thrust augmentation, the (a) in formulas (83) to (88) is replaced by $(a + a')$, hence, by $a(1 + \mu)$, after which

$$\frac{w_e}{V} = \sqrt{1 + \frac{2\eta_{th}q}{a(1 + \mu)}} \quad (100)$$

$$T = ma(1 + \mu)V \left[\sqrt{1 + \frac{2\eta_{th}q}{a(1 + \mu)}} - 1 \right] \quad (101)$$

$$\eta_p = \frac{a(1 + \mu)}{\eta_{th}q} \left[\sqrt{1 + \frac{2\eta_{th}q}{a(1 + \mu)}} - 1 \right] \quad (102)$$

$$\eta_g = \frac{a(1 + \mu)}{q} \left[\sqrt{1 + \frac{2\eta_{th}q}{a(1 + \mu)}} - 1 \right] \quad (103)$$

In these equations q always designates the parameter L/V^2 and η_{th} represents the thermal efficiency of the complete rocket-augmenter unit.

The calculation of this thermal efficiency η_{th} is predicated on the knowledge of the exhaust temperature of the assumedly homogeneous mixture ejected by the thrust augmenter. This involves the use of the equations of the operation of gas augmenters, that is to say, the equations established for the explosive rocket with thrust augmentation, which are to be used in the following.

36. Comparison with the true rocket

To evaluate the importance of the rocket with thrust augmentation, it is necessary and sufficient to compare it with the corresponding true rocket, that is, the true rocket that includes an identical generator of burnt gas and operates in the same conditions, but which, in addition, is fitted with a more or less different expansion nozzle, since in the case of the true rocket, this nozzle expands the burnt gases directly up to the surrounding pressure.

This nozzle will be, besides, identical if the mixer of the rocket with thrust augmentation operates at a pressure p_1 equal to the outside or atmospheric pressure p_a , that is, if the particular case involves the rocket with thrust augmentation characterized by

$$\lambda_1 = p_1/p_a = 1.$$

The study is restricted to a comparison of the over-all efficiency η_g of the rocket with thrust augmentation and the efficiency η_g'' of the corresponding true rocket, the corresponding quantities of the latter being distinguished by a double accent.

It will be remembered that

$$\eta_g = \frac{a(1 + \mu)}{q} \left[\sqrt{1 + \frac{2\eta_{th}q}{a(1 + \mu)}} - 1 \right] \quad (104)$$

and

$$\eta_g'' = \frac{a}{q} \left[\sqrt{1 + \frac{2\eta_{th}''q}{a}} - 1 \right] \quad (88)$$

also that

$$Q = \frac{\eta_{th}q}{a} = \frac{\eta_{th}L}{aV^2} \quad Q'' = \frac{\eta_{th}''q}{a} = \frac{\eta_{th}''L}{aV^2} \quad (105)$$

It is a question of studying the ratio

$$X = \frac{\eta_g}{\eta_g''} = (1 + \mu) \frac{\sqrt{1 + \frac{2Q}{1 + \mu}} - 1}{\sqrt{1 + 2Q''} - 1} \quad (106)$$

which depends only on Q , Q'' , and μ .

On the other hand, it is easily verified that by reason of the energy losses in the mixer of the augments (K > 0), Q must be smaller than Q''

$$Q < Q'' \quad (107)$$

Equation (106) is the cause of the superiority of the rocket with thrust augmentation over the corresponding true rocket, that is, for which $X > 1$; it is sufficient that

$$Q > Q_2 = \frac{Q'' + \mu \left[\sqrt{1 + 2Q''} - 1 \right]}{1 + \mu} \quad (108)$$

The two inequalities are compatible because obviously $Q_2 < Q''$. Starting from a given true rocket, its over-all efficiency can therefore be improved by the addition of a suitably designed thrust augmenter when the operating conditions $\lambda_1 = p_1/p_a$, $\mu = a'/a$, can be realized in the mixer of the thrust augmenter so that the ratio $Q = Q'' \times \eta_{th}/\eta_{th}''$ becomes greater than Q_2 .

In order to determine whether this realization is effectively possible, it is necessary to know the effective variation of the coefficient k as function of the conditions at the inlet of the mixer for properly designed gas augmenters. Insufficient knowledge on this subject makes the subsequent study based on assumption obligatory. Thus the procedure to be followed is the same as for the solid-fuel rocket; the study is limited to the specific case of the thrust augmenter characterized by a mixer at atmospheric pressure ($\lambda_1 = 1$). For the eventual effect of a modification of the pressure in the mixer ($\lambda_1 \neq 1$), the reader is referred to the considerations developed in article 20.

37. Specific case - mixer at atmospheric pressure

It should be noted that the formulas used in article 19 to link η_{th} to η_{th}'' are not directly applicable to the present case by reason of the difference in the thermodynamic cycles which concern, on the one hand, the solid-fuel rocket and, on the other, the generator of the rocket under consideration.

But, in the specific case of $\lambda_1 = 1$, it is easy to link η_{th} to η_{th}'' so that it can be indicated.

By definition, the thermal efficiency of the rocket with thrust augmentation, assumedly adiabatic, is

$$\eta_{th} = 1 - \frac{a(1 + \mu) C_m(T_e - T_a)}{L} \quad (109)$$

C_m designating the mean value between T_e and T_a of the specific heat of the gases delivered by the augmenter, the mean specific heat which is equally assumed valid for the burnt gases up to the exhaust temperature T_e'' of the true reference rocket.

This approximation is much more legitimate than in the case of the explosive rocket, because in this instance the burnt gases contain a considerable proportion of air and the exhaust temperatures are relatively low.

In these conditions the efficiency η_{th}'' of the true reference rocket is, by definition and since the rocket is always assumed adiabatic:

$$\eta_{th}'' = 1 - \frac{aC_m(T_e'' - T_a)}{L}$$

On the other hand, the application of the principle of the conservation of energy to the mixer of the thrust augments tube, also assumed adiabatic, results in

$$aC_m T_e'' + \mu aC_m T_a - (1 + \mu)aC_m T_e = (1 + \mu)a \frac{w_e^2}{2} - a \frac{w_e''^2}{2} - \mu a \frac{v^2}{2}$$

By definition of the coefficient k_1 comprised between zero and unity, which characterizes the kinetic energy losses in the mixer of the thrust augments, we get

$$(1 + \mu)a \frac{w_e^2}{2} - a \frac{w_e''^2}{2} - \mu a \frac{v^2}{2} = -k_1 a \left[\frac{w_e''^2}{2} + \mu \frac{v^2}{2} \right]$$

and lastly also

$$a \frac{w_e''^2}{2} = \eta_{th}'' L$$

With due regard to the preceding equations, T_e can be computed, as function of T_e'' , μ , and k_1 and which entered in (109) finally gives

$$\eta_{th} = \eta_{th}'' - k_1 \left(\eta_{th}'' + \frac{\mu a}{2q} \right) \quad (110)$$

Introducing this value in the expression (106) of X , the latter assumes for $\lambda_1 = 1$ the particular form

$$X_1 = (1 + \mu) \frac{\sqrt{1 + \frac{2Q''}{1 + \mu} - \frac{k_1(2Q'' + \mu)}{1 + \mu}} - 1}{\sqrt{1 + 2Q''} - 1} \quad (111)$$

For a given true reference rocket, Q'' is determined and the coefficient k_1 (a particular value, for $\lambda_1 = 1$ of the coefficient of the kinetic energy losses in the mixer) depends only on μ .

The ratio X_1 is therefore a simple function of μ and, when the function k_1 is known, it is readily ascertained when X_1 exceeds unity and its maximum determined.

Conversely, since k_1 is actually an unknown function, it is possible to determine the limit which this coefficient must not exceed, if X_1 is not to exceed unity.

Designating, as before, this limit, which depends only on Q'' and on μ , by K_1 its value, taken from (111) where $X_1 = 1$, is

$$K_1 = \frac{2\mu}{1 + \mu} \frac{1 + Q'' - \sqrt{1 + 2Q''}}{\mu + 2Q''} \quad (112)$$

Its numerical value for different Q'' and μ is given in table XIX and plotted in figure 27.

Table XIX

Values of the Function $K_1(\mu)$ and its Maximum

for Different Values of Q''

	Value of K_1 for:							Maximum of K_1	
	$\mu = 0$	0.5	1	2	5	10	20	Value	For $\mu =$
$Q'' = 2$	0	0.113	0.153	0.171	0.142	0.099	0.061	0.171	2
5	0	.170	.244	.298	.298	.244	.170	.284	3.162
10	0	.208	.306	.389	.427	.389	.306	.431	4.47
50	0	.272	.405	.535	.650	.677	.650	.675	10
100	0	.289	.431	.573	.706	.752	.751	.760	14.14

The maximum is reached at

$$\mu = \sqrt{2Q''}$$

and has the value

$$(K_1)_{\max} = \left[\frac{\sqrt{1 + 2Q''} - 1}{1 + \sqrt{2Q''}} \right]^2 \quad (113)$$

In order to appreciate the significance of the above numerical values given to parameter Q'' , table XX and figure 28 indicate the value of V (m/sec) corresponding to these values for the previously specified type of kerosene, where a and η_{th} have the extreme values considered as comprising the practical range, namely:

$$a = 15 \quad \text{and} \quad a = 75$$

$$\eta_{th}'' = 0.2 \quad \text{and} \quad \eta_{th}'' = 0.6$$

Table XX

Speed V for Several Values of a , η_{th}'' , and

$$Q'' = \eta_{th}'' L / a V^2$$

		$Q'' = 2$	5	10	50	100
$a = 15$	$\eta_{th}'' = 0.2$	$V = 552$	349	247	110.3	78.3
	0.6	955	605	428	191	135
$a = 75$	$\eta_{th}'' = 0.2$	$V = 246$	156	110	49.3	35
	0.6	426	270	191	85.5	60.5

In order to compute the practical possibilities of the rocket with thrust augmentation, several specific examples are analyzed.

Consider two cases of the true rocket corresponding to the practically extreme values of the dilution a and to the corresponding maximum thermal efficiency η_{th}'' , that is, $a = 20$, $\eta_{th}'' = 0.5$ and $a = 75$, $\eta_{th}'' = 0.3$.

Visualize, on the other hand, two values of the speed V identical to those considered previously for the explosive rocket with thrust augmentation, namely: $V = 112$ m/sec, = 402 km/h, and $V = 224$ m/sec = 804 km/h.

To these four particular cases taken for examples, there correspond the following values of Q'' :

$$\begin{array}{llll} Q'' = 91.7 & \text{for } a = 20, & \eta_{th}'' = 0.5 & V = 112 \\ Q'' = 22.9 & & & V = 224 \\ Q'' = 14.7 & \text{for } a = 75, & \eta_{th}'' = 0.3 & V = 112 \\ Q'' = 3.68 & & & V = 224 \end{array}$$

On plotting the values of k_1 corresponding to the values 1, 1.5, and 2 of the ratio $X_1 = \eta_g/\eta_g''$ against μ , the values of k_1 for the four cited Q'' are those represented in table XXI and illustrated in figures 29 to 32.

Table XXI

Values of $k_1(\mu)$ for Different Values of X_1

		$\mu = 0$	2	5	10	15	20
$Q'' = 91.7$	$X_1 = 1$	0	0.564	0.701	0.744	0.748	0.741
	1.5	-1.148	.144	.459	.586	.623	.635
	2	-2.72	-.417	.146	.390	.472	.506
	2.5	-4.74	-1.12	-.235	.159	.296	.361
$Q'' = 22.9$	$X_1 = 1$	0	0.476	0.561	0.556	0.526	0.490
	1.5	-1.06	-.057	.306	.386	.386	.372
	2	-2.485	-.483	-.007	.180	.228	.241
$Q'' = 14.7$	$X_1 = 1$	0	0.432	0.493	0.469	0.429	0.393
	1.5	-1.023	.019	.237	.296	.292	.277
	2	-2.39	-.503	-.066	.099	.141	.152
$Q'' = 3.68$	$X_1 = 1$	0	0.254	0.240	0.187	0.149	0.125
	1.5	-.88	-.108	.027	.0545	.0516	.0475

The ratio X_1 obtainable by thrust augmentation according to the μ value given by its corresponding volume is contingent upon the possibility of plotting on each of these diagrams the corresponding $k_1(\mu)$ curve characterizing the real mixer of the rocket with the thrust augmentation. As for the explosive rocket with thrust augmentation, the curve $k_1(\mu)$ which characterizes the real thrust augmentation can only be imagined.

In spite of the uncertainty of such speculation, it is assumed that the curve in question can be represented, in figures 29 to 32, by the curves C_1, C_2, C_3, C_4 , which are progressively toward the right to allow for the fact that the difference in jet velocities at the mixer entrance of the thrust augments decreases for each case.

On these premises, it is seen that the maximum of ratio X_1 corresponds to the points M_1, M_2, M_3 , and M_4 for which

$M_1:Q'' = 91.7$	$\mu = 5.6$	$k_1 = 0.2$	$X_1 = 2$
$M_2:Q'' = 22.9$	$\mu = 6.3$	$k_1 = 0.2$	$X_1 = 1.77$
$M_3:Q'' = 14.7$	$\mu = 5.5$	$k_1 = 0.14$	$X_1 = 1.71$
$M_4:Q'' = 3.68$	$\mu = 3.9$	$k_1 = 0.07$	$X_1 = 1.37$

These values of X_1 are, it is repeated, purely hypothetical and probably optimistic. Nevertheless, they are utilized for illustrative purpose.

They involve the following consequences for the four chosen examples: (Only the results of the calculations are given here.)

(1) $Q'' = 91.7$. - The true rocket having a thermal efficiency $\eta_{th}'' = 0.5$, a propulsive efficiency $\eta_p'' = 0.137$, and an over-all efficiency of $\eta_g'' = 0.0695$; the efficiencies for the best rocket with thrust augmentation and at a speed of 402 km/h will shift to

$$\eta_{th} = 0.396 \quad \eta_p = 0.346 \quad \eta_g = 2 \times \eta_g'' = 0.137$$

(2) $Q'' = 22.9$. - This case is the same as the one before except that speed of propulsion, supposedly doubled, is 804 km/h.

The true rocket is characterized by:

$$\eta_{th}'' = 0.5 \quad \eta_p'' = 0.255 \quad \eta_g'' = 0.1275$$

and the best rocket with thrust augmentation by

$$\eta_{th} = 0.386 \quad \eta_p = 0.585 \quad \eta_g = 1.77\eta_g'' = 0.226$$

(3) $Q'' = 14.7$. - The true rocket at 402 km/h speed is characterized by

$$\eta_{th}'' = 0.3 \quad \eta_p'' = 0.307 \quad \eta_g'' = 0.092$$

and the best rocket with thrust augmentation by

$$\eta_{th}'' = 0.250 \quad \eta_p = 0.629 \quad \eta_g = 1.71 \quad \eta_g'' = 0.157$$

(4) $Q'' = 3.68$. - This case is the same as case 3 except for the propulsive velocity of 804 km/h. The true rocket is characterized by

$$\eta_{th}'' = 0.3 \qquad \eta_p'' = 0.515 \qquad \eta_g'' = 0.1545$$

and the best rocket with thrust augmentation by

$$\eta_{th} = 0.268 \qquad \eta_p = 0.792 \qquad \eta_g = 1.37 \qquad \eta_g'' = 0.212$$

The foregoing data indicate that, if these assumptions can be realized, the particular rocket with thrust augmentation considered above ($\lambda_1 = 1$) is advantageous. It offers the possibility of lowering the speed of propulsion beginning at which the propeller with direct reaction gives an over-all efficiency comparable with that of the engine-propeller system. The speed at which both systems are equivalent is of the order of 500 to 800 km/h, depending upon the particular case.

Starting with a true rocket of lower thermal efficiency, the rocket with thrust augmentation thus offers the possibility of obtaining a propeller whose efficiency exceeds that of a rocket with thrust augmentation that corresponds to a true rocket of higher thermal efficiency, the speed of propulsion remaining, of course, the same.

Thus at 402 km/h the rocket with thrust augmentation corresponding to the true rocket characterized by $\eta_{th}'' = 0.3$ and $a = 75$ (case 3) will have an over-all efficiency of $\eta_g = 0.157$, hence be superior to that of the rocket with thrust augmentation corresponding to the true rocket characterized by $\eta_{th}'' = 0.5$ and $a = 20$ (case 1), an efficiency that reaches only $\eta_g = 137$.

So, in the first of these cases, the rocket with thrust augmentation is able to reach $\eta_{th} = 0.25$ for a total volume equal to $a(1 + \mu) = 75(1 + 5.5) = 487$, against $\eta_{th} = 0.396$ in the first case, which is superior to the former, but with a volume of only $20(1 + 5.6) = 132$. The increase in volume outweighs the decrease in thermal efficiency, that is, by the speed of ejection of the gases at the augments outlet.

Moreover, the whole problem of the rocket with augmentation hinges on knowing whether this arrangement, which obviously lowers the thermal efficiency with respect to the corresponding true rocket, will make it possible to increase the total volume enough to improve the over-all efficiency.

In this respect, the following problem may be put.

Imagine a liquid-fuel rocket operating at constant pressure and at a previously determined compression ratio. The efficiency of expansion and compression being assumedly fixed, the thermal efficiency of this true rocket depends upon the dilution a of the combustible mixture. By progressively increasing the dilution, the efficiency is reduced and finally nullified.

Instead of effecting the preliminary dilution of the combustion gases by going through the cycle of the entire mixture in rocket, the exhaust gases of the true rocket can be diluted by the air trapped by a thrust augments tube. In this case also the thermal efficiency of the system, that is, of the rocket with thrust augmentation, is reduced.

From the point of view of over-all efficiency, it may be interesting to find out if and in what conditions the extrinsic dilution is more advantageous than the intrinsic.

38. Intrinsic and extrinsic dilution

The problem involved is simplified by the following approximations. The specific heat of air is compared to that of the burnt gases and their variation with the temperature disregarded. This assumption is so much more justified as the dilutions involved are greater and the temperatures are therefore lower.

Supposing that the expansion and compression nozzles, as well as the mixer of the thrust augments (which operates at atmospheric pressure), are actually adiabatic, and that λ is the ratio of the primary compression.

Two rockets are considered: One consumes, per unit mass of fuel, the air mass a in the true rocket and the mass μa captured by the thrust augments; the other consumes the same total mass of air $a(1 + \mu)$ and has no thrust augments.

The quantities of the second rocket are indicated by a triple accent and the corresponding gas phases (compared to air at constant specific heat C) at induction, terminal compression, after combustion and completed expansion by the subscripts a , b , c , and d . Lastly, assume that the combustion (effected in both cases at pressure λp_g) is complete.

It is a question of comparing the thermal efficiencies

$$\eta_{th} \text{ and } \eta_{th}'''$$

of the rocket fitted with thrust augments and the pure rocket of the same total volume.

According to previous arguments, the efficiency of the rocket with thrust augmentation is

$$\eta_{th} = \eta_{th}'' - \frac{aQ_m}{L} \quad (114)$$

η_{th}'' denoting the thermal efficiency of the corresponding true rocket obtained by elimination of the thrust augments and (aQ_m) the amount of heat equivalent to the energy of the viscosity and the friction in the mixer of the thrust augments, a quantity referred to unit of mass of fuel consumed. The preceding equation follows immediately from (61) where $\lambda_1 = 1$.

The efficiency η_{th}'' can be put in the form

$$\eta_{th}'' = \frac{a \left[C(T_c - \lambda^n T_a)(1 - \lambda^{-n}) - Q_a - Q_d \right]}{L} \quad (115)$$

n designating the coefficient $\frac{\gamma - 1}{\gamma}$ characterizing the variation of temperature in the irreversible adiabatic transformations, Q_a and Q_d denoting the quantities of heat equivalent to the energy of viscosity and friction during compression and expansion in the rocket, quantities referred to unit mass of air of the cycle.

On the other hand, the thermal efficiency η_{th}''' of the second true rocket with the same total volume as the rocket with thrust augmentation, is put in the form

$$\eta_{th}''' = \frac{a(1 + \mu) \left[C(T_c''' - \lambda^n T_a)(1 - \lambda^{-n}) - Q_a''' - Q_d''' \right]}{L} \quad (116)$$

quantities Q_a''' and Q_d''' having the same significance as Q_a and Q_d and quantity Q_a''' can also be identified with Q_a when the compressors of the two rockets have the same efficiency with respect to the reversible adiabatic compression: It is admitted here by putting $Q_a''' = Q_a$.

Next, the combustion temperatures T_c''' and T_c in the two compared rockets are evaluated.

In each case, since the combustion is assumed complete,

$$aC(T_c - T_b) = L \quad (117)$$

$$a(1 + \mu)C(T_c''' - T_b''') = L \quad (118)$$

On the other hand, the temperatures T_b and T_b''' at the end of compression are identical, the compressors operating with the same compression ratio and the same specific efficiency. Hence

$$T_b = T_b''' = T_a \lambda^n + \frac{Q_a}{C} \quad (119)$$

With (119) taken into consideration, the values of T_c and T_c''' computed by (117) and (118) can be used to explain formulas (115) and (116). Finally there is afforded for the rocket with thrust augmentation

$$\eta_{th} = (1 - \lambda^{-n}) - \frac{a}{L} \left[\frac{Q_a}{C} \lambda^{-n} + Q_d + Q_m \right] \quad (120)$$

and for the true rocket with the same total volume:

$$\eta_{th}''' = (1 - \lambda^{-n}) - \frac{a(1 + \mu)}{L} \left[\frac{Q_a}{C} \lambda^{-n} + Q_d''' \right] \quad (121)$$

These formulas bring out the reducing effect of the thermal efficiency $(1 - \lambda^{-n})$ of the theoretical thermodynamic cycle, of the work of viscosity and friction (in absolute value) in the various parts of the systems.

These approximate formulas show that, for the extrinsic dilution to be more beneficial than the intrinsic dilution, that is, η_{th} superior to η_{th}''' , it is necessary and it suffices that

$$Q_m < (1 + \mu) \left[\frac{Q_a}{C} \lambda^{-n} + Q_d''' \right] - \left[\frac{Q_a}{C} \lambda^n + Q_d \right] \quad (122)$$

This relation states that the work of friction and viscosity in the mixer referred to unit mass of motive fluid in the thrust augments must be less than the difference in the energies of the same nature during the compression (the latter multiplied by the factor λ^{-n} less than unity) and during the expansion in the pure rocket, and in the rocket with thrust augmentation, the said energies being themselves referred to unit of mass of fluids in the cycle.

This precise conclusion is important. There is nothing to assert positively that it can not be achieved. It is not contradictory to the Carnot-Clausius principle, and the theory of the viscosity of gases, in its present state, is insufficient to elucidate the question by theoretical considerations. The possibility of satisfying the condition (122) must be left to experiment.

It may be added that in the foregoing the study was limited to the particular case of thrust augmenters with mixer operating at atmospheric pressure and that, in general, this condition need not necessarily correspond to the maximum over-all efficiency of the rocket fitted with thrust augmentation.

C. SUMMARY AND CONCLUSIONS OF THE STUDY ON

DIRECT AND AXIAL JET PROPULSION

39. Recapitulation of results

The principal results may be summed up as follows:

(1) The explosive rocket is unsuitable as normal system of propulsion for aircraft by reason of its excessive consumption by weight and which results from its low over-all efficiency up to very high speeds, and to the low heat value of explosives.

(2) Only the rocket using an ordinary fuel, for example, liquid, with high heat value can furnish a normal means of propulsion, and then, only at very high speeds.

(3) To improve the over-all efficiency of such a rocket or to lower the speed beginning at which its efficiency becomes important, it is recommended to increase the dilution of the expelled gases without unduly lowering the thermal efficiency of the complete system.

(4) To this end, either the intrinsic dilution, that is, the increase in the proportion of air in the combustible mixture subjected to combustion after compression, may be considered or else the extrinsic dilution, that is, the entrainment, by the burnt gases, of fresh air captured from the outside by means of a thrust augments tube.

(5) At the present stage of development of heat engines, the pure rocket using kerosene, with great dilution and at relatively low pressures and temperatures of operation, appears to give a somewhat greater over-all efficiency than that of a rocket with weak dilution and high compression, a machine which should preferably be of the reciprocating type.

(6) The best solution of the pure rocket using kerosene appears to be supplied by the turborocket, for which the great dilutions and the low pressures and temperatures occasion no great difficulty in realization.

(7) The rocket with augmentation, realizing the extrinsic dilution, appears susceptible, in certain cases, to a higher over-all efficiency than the corresponding true rocket.

(8) In order to elucidate this point and to enable the prediction of the best thrust augments the systematic study of gas augmenters is imperative.

(9) This study, which is important in numerous applications in many fields, must rest largely upon the determination of the coefficient k , which characterizes the kinetic energy losses in the mixer of a gas augments. This coefficient k depends on:

The nature of the secondary and the primary gases and their physical properties (conductivity, specific heat, internal and contact friction and viscosity)

The state of these gases (pressure, temperature, velocity) at entrance in the mixer of the thrust augments

Their corresponding proportions (corresponding volume of augments tube)

Lastly, disposition of the augments tube (concentric, inserted, fragmentary jets, etc.)

The experimental and systematic study of the coefficient k , even when restricted to the essential characters of this coefficient, must supply all the elements of evaluation, with the considerations and calculations developed in the foregoing but until then it is necessary to maintain the hypothetical character of the previous conclusions.

Lastly, it should be noted that the present study ties in with the ideas developed by Rateau in his theory of thrust augmenters, that is, in the adoption of the concept of a mixer producing an ultimately homogeneous mixture of primary and secondary fluids at constant pressure, and uniform temperature and velocity. One of the very first concerns of such an investigation should be to check the basic principle of this conception, which is still insufficiently established in our opinion.

40. Comparison of rocket and engine-propeller system

The over-all efficiency of the engine-propeller system considered as an isolated system as for the rocket, that is, by disregarding all aerodynamic interactions between propulsion and the propelled system, is

$$\eta_g = \eta_{th} \times \eta_h \quad (123)$$

η_h denoting the propulsive efficiency of the propeller and η_{th} the effective thermal efficiency of the engine (at the propeller shaft), which efficiencies are defined in the usual manner.

In the air, the propeller constitutes an indirect jet propeller. In fact it is a question of giving the ambient air a downstream recoil motion. The simplest way to describe it is by comparing it to the so-called Froude propeller, according to which the propeller acts on a cylindrical and limited stream of air to which is communicated a uniform and axial speed of recoil. Such a propeller may conceivably be realized by means of two coaxial propellers rotating in opposite direction behind each other and on the inside of a more or less extended lateral envelope which forms the boundary of the air stream on which the propeller acts direct.

The significance of the concept of the Froude propeller rests on the fact that, for equal thrust at a propulsive velocity and a given propeller diameter, this propeller has a slightly higher efficiency than a good pusher propeller of the usual type. The efficiency of this propeller is in this respect often regarded as the upper limit of the propulsive efficiency attainable by means of a propeller for a given thrust intensity. While not exactly correct from the theoretical point of view, this mode of viewing it is nevertheless justified by experiment and for the ordinarily employed propellers.

In any case, when S_h denotes the upstream section of the air stream stirred up and pushed back by the propeller, a section that may be compared to the swept-disk area of a propeller ($S_h = \frac{\pi D^2}{4}$, D = propeller diameter), and ρ_a signifies the density of the surrounding air, an (nondimensional) aerodynamic thrust coefficient of the propeller can be defined by the relation

$$c_t = \frac{T}{\rho_a S_h \frac{V^2}{2}} \quad (124)$$

and it is easily established that the efficiency η_h of the Froude propeller, taken in this instance as the ideal propeller type, is linked to the preceding coefficient by the relation

$$\eta_h = \frac{4 + c_t}{4} \quad (125)$$

With m_A denoting the volume of air stirred up and forced back by this propeller (A being the specific volume referred to unit mass of fuel consumed by the engine),

$$m_A = \rho_a S_h V \quad TV = \eta_h m \eta_{th} L$$

η_{th} = the thermal efficiency of the engine. Hence

$$c_t = 2\eta_h \frac{\eta_{th} L}{AV^2} \quad (126)$$

Putting

$$Q = \frac{\eta_{th} L}{AV^2} \quad (127)$$

and eliminating c_t between (125) and (126) leaves

$$\eta_h = \frac{1}{Q} \left[\sqrt{1 + 2Q} - 1 \right] \quad (128)$$

This relation is identical to (90) which gives the propulsive efficiency of a rocket with relatively high air consumption a . The only difference is that the specific volume of air a of the rocket is replaced here by the specific volume A of the Froude propeller.

Equation (128) shows that η_h increases continuously and tends toward unity when Q tends toward zero, that is, when η_{th} , L , and V are given, then A increases indefinitely.

This remark suffices to demonstrate, it may be said in passing, the superiority of the principle of geared-down propellers.

For the ordinary airplane propellers adapted in the best conditions, the propulsive efficiency η_h is inferior to that of the Froude propeller, the corresponding coefficient of reduction being practically

constant and of the order of 0.85, so that the efficiency of these propellers can be expressed by the simple formula

$$\eta_h = \frac{0.85}{Q} \left[\sqrt{1 + 2Q} - 1 \right] \quad (129)$$

Compare now the engine-propeller system whose efficiency is to be summarily evaluated, with the direct reaction propeller, that is, the rocket.

It has been shown that the explosive rocket cannot be envisaged as normal means of propulsion, especially by reason of its absolutely prohibitive fuel consumption by weight in aviation.

Hence, only the liquid-fuel rocket is considered.

For it to have the same over-all efficiency as the reference engine-propeller system at the same forward speed, it is necessary and sufficient that, according to (88) and (129) and with identical fuel in both cases:

$$0.85 A \left[\sqrt{1 + 2 \frac{\eta_{th} L}{AV^2}} - 1 \right] = a' \left[\sqrt{1 + \frac{2\eta_{th}' L}{a' V^2}} - 1 \right] \quad (130)$$

the accented quantities refer to the rocket.

In case of equality of the thermal efficiency $\eta_{th} = \eta_{th}'$ for the two engine propeller systems compared, this condition would be equivalent to:

$$a' = 0.85A$$

This condition would become $a' = A$ if the propeller were a perfect Froude propeller. The latter being ordinarily considered as the ideal type of pusher propeller, the following theorem holds true:

For a rocket to be equivalent to an ideal engine-propeller system, at the same speed and for the same fuel, it is sufficient

- (1) That the thermal efficiency of both systems be the same
- (2) That the air intake of the rocket be of the same cross section as the (upstream) section of the air activated by the propeller

The disposition of the two equivalent systems is shown diagrammatically in figure 33.

At ordinary speeds of propulsion, the rocket cannot pretend to furnish, with a suitable thermal efficiency, specific volumes comparable to those of propellers.

But at speeds in excess of 1000 km/h, the rocket is comparable to the engine-propeller system. At those speeds, in fact, the propeller appears to undergo an appreciable drop in efficiency which ultimately results in the superiority of the rocket.

The addition of a thrust augmenter holds out the promise of obtaining, always at high speeds, an appreciable advantage in favor of rocket propulsion, but the fact cannot be established until certain experiments, never undertaken to our knowledge, have been made.

The subject is concluded with a remark about the relationship existing between the previously studied types of rockets and the classical propeller-engine system.

Visualize an internal combustion turbine driving an ordinary propeller, the turbine comprising an air turbocompressor, combustion chambers, and an expansion turbine; a' is the specific volume of the turbine and A that of the propeller which it drives.

In the most favorable operating conditions, all the expansion energy of the burnt gases is converted in the turbine into mechanical energy, part of which is consumed by the compressor situated in the turbine and part by the propeller. The speed of exhaust of the burnt gases is negligible at the turbine exit. The turbine being supposed to have several runners, it is assumed that the runners situated downstream from the upper stages where the energy necessary to drive the compressor is realized, are suppressed, and that the expansion of the exhaust gases is accomplished in a well-designed nozzle at the outlet of the thus-truncated turbine. The efficiency of the latter being greater than that of the stages suppressed in the turbine, there is obtained a jet of gas whose kinetic energy at ejection is, other things being equal, a little higher than the energy transmitted to the propeller. The thermal efficiency of the system is slightly improved and, this time, it is the gaseous jet leaving the thus formed turborocket that furnishes the propeller force, by reaction.

If the volume of this jet is comparable to that of the air stirred up and forced back (mechanically) by the original propeller, it is immediately apparent that the corresponding speeds are comparable also, as well the corresponding propulsive and over-all efficiencies.

The last runners of the original turbine and the propeller are merely designed to transfer the energy of the final part of the expansion of the burnt gases to an external mass of air to which this energy was transmitted in the form of kinetic energy.

At very high speeds, the superiority of the rocket over the engine-propeller system increases and it is readily apparent that the suppression of the propeller and the corresponding stages in the expansion turbine afford, for the realization of the engine-propeller system, a fortunate mechanical simplification and a substantial saving in weight.

This is, moreover, one of the reasons which confer particular and primary interest to the problem of turbomachines with internal-combustion in the range of propulsion at very high speeds, that is, above 800 to 1000 km/hour.

P A R T I I

HELICOIDAL REACTION PROPULSION SYSTEMS

Chapter I Jet Propeller

41. Definition of the systems under consideration

The systems involved consume a fuel supplied on board the airplane and take from the surrounding atmosphere the air necessary for combustion, to which a corresponding excess can be added.

They are characterized by the fact that the ejection of the cyclic fluids, more or less converted by the combustion, is effected by one or more nozzles impelled by a helicoidal movement. The type of these systems is represented diagrammatically by figure 34.

The air, taken from the outside atmosphere, enters the machine through a fixed and axial opening A facing forward.

On passing through the machine this air and the fuel are subjected to certain changes in their physical and chemical state comprising three essential phases: compression, combustion, and expansion. This thermodynamic cycle is accomplished in part in a heat engine¹¹ M and in part in a rotating system C which comprises the exhaust arrangement.

The rotating system is connected to the engine M and can receive from or supply energy to the latter. (In the first case, the engine M is then, strictly speaking, a receiver.)

System C comprises the exhaust which is effected by orifices oriented perpendicular to their helicoidal trajectory and rearward. It drives a propeller¹² H with which it even may be identical as in the case of figure 34.

It is immediately apparent that this general scheme comprises the following particular cases:

¹¹The term "engine" is taken here in a very general, and so to speak, algebraic sense; the said engine may be actually a receiver, that is, receive energy from the outside instead of supplying energy to it.

¹²It could be assumed that several propellers, equivalent to the single propeller considered here for the evident object of simplification, are involved.

(a) Rocket with direct reaction.- To obtain the latter, it is sufficient to eliminate the propeller H and assume the rotating system C stationary. The engine M then supplies no energy to the outside and the fixed exhausts logically take their place downstream from the engine and to the rear of the arrangement, restoring the normal outline of the rocket. If the latter consumes no outside air (true explosive rocket), the air intake A must be removed.

(b) True jet propeller.- To obtain it, simply eliminate the engine M. The compression of the air is then accomplished entirely in the rotating arms that constitute the blades of the propeller, the tips of which are the burners which feed the exhaust nozzles. The propeller is then driven exclusively by the reaction of these nozzles, whence the adopted term "pure reaction propeller."

(c) Classical engine-propeller system.- To obtain this system, simply assume the diameter of the rotating system to be zero. The exhaust becomes then fixed again and the propeller is driven by the engine M in which the thermodynamic cycle is completed.

The subsequent operating formulas furnish a sort of general theory of propulsion systems with direct, indirect (that is, performed by a mechanical system in the surrounding medium, case of the propeller), and mixed reaction.

It should be noted that the exhaust jets of the rockets are assumed to face in the opposite direction of the absolute speed of these rockets. This is a simple and logical conception, very close to that which corresponds, in fact, to the best orientation, and which is to be discussed again in article 63.

42. Simplifying assumptions

These assumptions, intended to simplify the problem, which introduce but a negligible error, are as follows:

(1) The pressure in the straight sections S_a and S_e of the air intake and the exhaust is uniform and equal to p_a .

(2) The state of the fluid in these sections is homogeneous and steady and their speed uniform, when normal operation is established.

(3) Normal operation, periodic theoretically (especially with reciprocating engine), is compared to a steady state, which amounts to characterizing the quantities considered by their average values during one cycle, once the steady state is reached.

(4) The resultant of the actions of the ambient fluids on the outside wall of a revolving rocket is reduced to a resistance \bar{R}_e opposite to the absolute speed \bar{W}_e of the center of the corresponding exhaust opening.

(5) At the point where the cyclic bodies pass from engine M to the rotating system C , the speed of these bodies is parallel to the axis of rotation.

43. Definition and symbols

m = mass of fuel consumed in unit time

a = corresponding consumption of air or ratio of air mass to fuel consumed in the same time

V = axial	}	speed of the revolving rockets (measured in the center of the exhaust orifice)
U_e = peripheral		
W_e = resultant		

β_e = angle of W_e and of V

w_e = corresponding speed of exhaust in the section S_e

$w_a = V$ = corresponding speed of air at its entry in the mouth S_a of the air intake

C_m = mechanical energy supplied by the engine to the propeller (by means of the rotating system) and referred to unit mass of fuel consumed

To define the net thrust supplied by the propulsion system as well as the resistance of its various elements accurately, the system is decomposed in three parts:

(1) The central body terminating in front in the air intake A

(2) The propeller blades limited by the straight sections or blade profiles which correspond, on the one hand, to the passage of the blade in the central body (junction with the faired hub), and on the other, to the junction of the blade with the rocket located at its tip

(3) The revolving rockets to the number of n , arranged symmetrically around the axis of rotation

The central body or envelope and the rockets are fitted with openings S_a and S_e and the pressure existing in it is compared to the outside pressure p_a . These openings introduce a special difficulty for the evaluation of the resistances, as already pointed out on the subject of rockets (article 6).

Let

R_m be the resultant of the aerodynamic forces on the outside of the central body, the resultant assumed oriented along the propeller axis and opposite to speed V

\bar{T}_h the axial resultant of the aerodynamic forces on the propeller blades, or the propeller thrust

\bar{R}_e the resultant of the aerodynamic forces on the outer surface of a rotating rocket, a resultant assumed oriented along the axis of the rocket and opposite to its absolute forward speed W_e

T' the effect of the thrust which the system can transmit to the outside

The resistance of a body with open outside surface (the case of the central body and of the rotating rockets) is the projection on the speed of displacement of this body counted positive in the opposite direction of the said speed, of the resultant:

Of the tangential force of the ambient fluid on the outside surface in question, and of the normal forces of the same fluid on the same surface, uniformly reduced forces of the value p_a of the general pressure of the outside medium

Then the resistance of the central body is the force

$$R_m' = R_m - p_a S_a' \quad (131)$$

opposite to speed V , the resistance of a rotating rocket, the force

$$R_e' = R_e - p_a S_e \quad (132)$$

opposite to speed W_e and directed along the axis of the rocket. The force

$$T = T' + R_m' \quad (133)$$

counted in the direction of the speed V , the force which represents the total resistance of the central body and of the external systems that can balance the thrust of the system of propulsion, is termed the real thrust of the system.

This convention is logical to the extent that the central body can be developed so that it constitutes the body itself of the propelled (or towed) system, as in the case of an airplane with its engine mounted in the fuselage. The term R_m' includes the aerodynamic interference of the propeller on the central body. Reciprocally, the propeller thrust T_h comprises the interference of the central body and the towed system exerted on the propeller.

With C_h denoting the propeller torque, counted positive in the direction opposite to the speed of rotation ω_h of the propeller, the propulsive efficiency η_h of the propeller is

$$\eta_h = \frac{T_h V}{C_h \omega_h} \quad (134)$$

This definition agrees with that usually employed for defining the efficiency of an isolated propeller, but in this instance, the terms T_h and C_h comprise the interference of the outside system and, in addition, refer only to actions submitted by the propeller blades proper. The blade tips belong, in effect, to the revolving rockets if the system comprises it, while the blade roots or the hubs belong to what is called the central body.

In spite of these differences, the discountable values of efficiency η_h defined by (134) are, for equal diameter and axial and peripheral speeds, fairly close to those furnished by a complete propeller of the same type, assumed bare and isolated.

On the basis of these definitions, the study of operation of the system can be undertaken.

As before, energy, work, and heat quantities are assumed to be expressed per unit of mechanical energy; the numerical calculations in M.K.S. units.

44. Thermodynamic cycle - thermal efficiency

Fuel and air start from an initial state $a(p_a, T_a)$ where their respective speeds at entry in the system are V and zero.

Both the fuel and the air undergo a transformation which brings them, after passage in the engine \underline{M} , into an intermediate stage $i(p_i, T_i)$ which is assumed homogeneous and steady where their corresponding speed w_i is parallel to the axis of the rotating system and situated, for each gas particle, at a practically negligible distance from this axis. (This is equivalent to saying that the gases are conducted to the rotating system by an axial duct of rather reduced section, which is, naturally, the system called "jet propeller.")

The cycle is completed in the rotating system and brings the error to the exhaust state $e(p_e = p_a, T_e)$, likewise assumed homogeneous and steady, where their corresponding speed is w_e , in opposite direction to the speed W_e of the revolving rocket.

With \underline{U} and \underline{V} as the internal energy and the volume of the fuel-air mixture per unit mass of fuel, and $m\dot{Q}_{Rm}$ and $m\dot{Q}_{Rt}$ as the heat transfer per unit time of the engine \underline{M} and of the rotating system \underline{C} , the heat transfer of the whole to the outside is

$$m\dot{Q}_R = m\dot{Q}_{Rm} + m\dot{Q}_{Rt} \quad (135)$$

According to the definitions given in article 2, the effective energy \underline{C}_{eff} of the thermodynamic cycle referred to unit mass of fuel consumed is

$$\underline{C}_{eff} = (\underline{U} + p\underline{V})_a - (\underline{U} + p\underline{V})_e - \dot{Q}_R \quad (3)$$

and the thermal efficiency η_{th} of the system in question is

$$\eta_{th} = \frac{\underline{C}_{eff}}{\underline{L}} \quad (4)$$

The heat balance of the whole of the system, expressed by (3), can be divided in two parts, one of the engine \underline{M} , the other of the rotating system \underline{C} .

The application of the principle of the conservation of energy to the engine \underline{M} and to the bodies contained in it with respect to the axes moving at speed V and per unit of time gives the heat balance of the engine at

$$(\underline{U} + p\underline{V})_a - (\underline{U} + p\underline{V})_i - \dot{Q}_{Rm} = \underline{C}_m + (1 + a) \frac{i^2 w^2}{2} - a \frac{v^2}{2} \quad (136)$$

while taking into account the admitted steadiness of the extreme states a and i and bearing in mind that the engine supplies in unit time the energy $m\dot{C}_m$ (taken in algebraic value) to the rotating system \underline{C} .

Then the principle of the conservation of energy is applied, in the same conditions, to the rotating system C (propeller plus rockets).

In this application, the work of the aerodynamic actions supported by the propeller and the outside walls of the rockets with respect to the axes in translation must be taken into consideration. The kinetic energy involved is that of the corresponding motion with respect to the axes in question. Hence the heat balance of the rotating system is

$$(\underline{U} + p\underline{V})_i - (\underline{U} + p\underline{V})_e - \underline{Q}_{Rt} = -\underline{C}_m + \frac{1}{m} C_h \omega_h + \frac{n}{m} R_e 'U_e \sin \beta_e + \left[\frac{1+a}{2} (w_e^2 + U_e^2 - 2U_e w_e \sin \beta_e) - (1+a) \frac{w_i^2}{2} \right] \quad (137)$$

Equations (136) and (137) can serve to calculate \underline{C}_m (or w_i) and $C_h \omega_h$ (or w_e) when the intermediate state i and the final state e , as well as the heat loss due to external friction (\underline{Q}_{Rm} and \underline{Q}_{Rt}) of M and C, are fixed.

These are the relations to be used in the discussion of the thermodynamic cycles, whose realization may be visualized.

In certain problems, it will be expedient to fix, a priori, the value of \underline{C}_m . Therefore, we put

$$\underline{C}_m = h \underline{C}_{eff} = h \eta_{th} L \quad (138)$$

The coefficient h thus defined represents the part of the effective energy of the thermodynamic cycle given off in the form of mechanical energy by the engine M on the rotating system C with which it is assumed to be connected.

Adding (136) and (137), while taking (135) into consideration, gives

$$(\underline{U} + p\underline{V})_a - (\underline{U} - p\underline{V})_e - \underline{Q}_R = \frac{1}{h} C_h \omega_h + \frac{n}{m} R_e 'U_e \sin \beta_e + \left[\frac{1+a}{2} (w_e^2 + U_e^2 - 2U_e w_e \sin \beta_e) - a \frac{V^2}{2} \right] \quad (139)$$

which expresses the heat balance of the complete system, and which, with consideration to (3) and (4), that is, the definition of the thermal efficiency η_{th} , can be written as

$$\eta_{th}L = \frac{1}{m} C_h \omega_h + \frac{n}{m} R_e' U_e \sin \beta_e + \left[\frac{1+a}{2} (w_e^2 + U_e^2 - 2U_e w_e \sin \beta_e) - a \frac{V^2}{2} \right] \quad (140)$$

a relation which is applied later for calculating the corresponding speed of exhaust w_e .

45. Real thrust of the propulsion system

The application of the momentum theorem, in steady state and projected on the direction of speed V , to the system and to the cyclic bodies contained in it gives

$$-T' - (R_m - p_a S_a) + T_h - n(R_e - p_a S_e) \cos \beta_e = -m \left[(1+a) w_e \cos \beta_e - aV \right]$$

Taking the definitions (131), (132), and (133) into consideration, this relation gives the real thrust T in the form

$$T = T_h - nR_e' \cos \beta_e + m \left[(1+a) w_e \cos \beta_e - aV \right] \quad (141)$$

The first term of the second member represents the real propeller thrust (resultant of the indirect reaction caused by the motion of the latter in the outside fluid), while the third term represents the reaction due to the fluid jets leaving the system (helicoidal reaction of n rockets) or entering the system (resistant reaction of the air intake).

The resistance R_e' of a rocket, as defined by (132), can be expressed in the form

$$R_e' = c_{re} \frac{\rho_a}{2} S_e w_e^2 \quad (142)$$

with c_{re} denoting the coefficient of the aerodynamic resistance of this rocket, that is, the resistance of the rocket with respect to the

air density ρ_a , of the ambient fluid to the section S_e of the outlet orifice and to the $1/2$ of the square of the absolute speed W_e in the center of this orifice.

The total exhaust volume (by mass) is

$$m(1 + a) = n\rho_e S_e' w_e$$

ρ_e denoting the density of the ejected fluids, so that (142) can be written as

$$Re' = m \frac{1 + a}{2n} c_{re} \frac{\rho_a}{\rho_e} \frac{V^2}{w_e \cos^2 \beta_e} \quad (143)$$

The thrust T_h of the propeller is expressed by (134) as function of η_h and $C_h \omega_h$.

To evaluate the resistant energy $C_h \omega_h$ of the propeller the theorem of kinetic moments about the axis of rotation is applied to the rotating system \underline{C} (propeller plus rockets).

Multiplying the various moments by ω_h gives

$$mC_m - C_h \omega_h - nRe' \sin \beta_e U_e = -m(1 + a)(w_e - W_e) \sin \beta_e U_e$$

with the definition of C_m (138) and the geometric relations $U_e = V \tan \beta_e$, $W_e = V/\cos \beta_e$, this equation becomes

$$C_h \omega_h = mh\eta_{th}L - nRe' \sin \beta_e \tan \beta_e V + m(1 + a)(w_e \cos \beta_e - V) V \tan^2 \beta_e \quad (144)$$

The real thrust T , expressed by (141), becomes

$$T = m \left\{ \frac{h\eta_h \eta_{th} L}{V} + (1 + a)(1 + \eta_h \tan^2 \beta_e) V \left[\frac{w_e \cos \beta_e}{V} - \left(1 + \frac{c_{re}}{2} \frac{\rho_a}{\rho_e} \frac{V}{w_e \cos \beta_e} \right) \right] + V \right\} \quad (145)$$

in which h , η_h , η_{th} , $\tan \beta_e$, and w_e are the principal unknowns.

46. Corresponding speed of exhaust

By (139) in which $C_{h\omega_h}$ is replaced by its value given in (144)

$$(1 - h) \eta_{th} L = (1 + a) \frac{w_e^2}{2} - a \frac{V^2}{2} \left[1 + \frac{1 + a}{a} \tan^2 \beta_e \right] \quad (146)$$

This fundamental relation defines w_e when a , the corresponding air consumption of the system, η_{th} , the thermal efficiency of the complete system, L , the heat value of the fuel, h , part of the effective energy of the transferred thermodynamic cycle, in the form of mechanical energy from \underline{M} to \underline{C} , V , the speed of propulsion, $\tan \beta_e = \frac{U_e}{V}$, the inverse of the path of the helicoidal trajectory of the rockets, are known.

The parameters in (146) are made dimensionless, by putting

$$x_e = \frac{w_e}{V} \quad (147)$$

$$Q = \frac{\eta_{th} L}{a V^2} \quad (148)$$

$$\xi = \frac{1 + a}{a} \quad (149)$$

Then the relation (146) resolved with respect to x assumes the form

$$x_e = \sqrt{\frac{1}{\xi} + \tan^2 \beta_e + \frac{2(1 - h)Q}{\xi}} \quad (150)$$

The case $h = 0$ corresponds to the propeller driven exclusively by the reaction of the revolving rockets.

The case $h = 1$ corresponds to the classical engine-propeller system (without revolving rockets).

Equation (150) shows that, for all intermediate cases, that is, where h ranges between 0 and 1, x_e is greater than $\frac{1}{\xi} = \frac{a}{a + 1}$. This assures only that w_e is always greater than V , since $\frac{1}{\xi}$ is less than unity.

47. Propulsive efficiencies

The over-all efficiency η_g of the propulsive system is given by

$$\eta_g = \eta_{th} \eta_p \quad (8)$$

the propulsive efficiency η_p being defined by

$$\eta_p = \frac{TV}{m\eta_{th}L} \quad (9)$$

The real thrust T is given by (145). The substitution in (145) of the notations x_e , Q , and ξ defined by (147), (148), and (149) results in an expression which, entered in formula (9), gives, after reduction, the following expression of η_p

$$\eta_p = h\eta_h + (1 - h) \frac{2\xi}{\xi x_e^2 - (1 + \xi \tan^2 \beta_e)} \times \left\{ (1 + \eta_h \tan^2 \beta_e) \left[x_e \cos \beta_e - \left(1 + \frac{c_{re}}{2} \frac{\rho_a}{\rho_e} \frac{1}{x_e \cos \beta_e} \right) \right] + \frac{\xi - 1}{\xi} \right\} \quad (151)$$

This relation can be simplified, in general, by disregarding the term in c_{re} .

According to previous statements, c_{re} is a number which probably does not exceed the value 0.12.

The ratio ρ_a/ρ_e can, at most, reach a value of the order of 2 to 3, because in a device of appropriate thermal efficiency the exhaust occurs at an absolute temperature, at best, two or three times as high as the absolute ambient temperature, a limit reduced here also, since an appreciable kinetic exhaust energy is involved if a reaction effect is counted on.

The ratio $x_e = w_e/V$ is, in general, a number of the order of several unities when V remains below 200 to 250 m/sec.

Angle β_e should scarcely exceed 60° if V exceeds 100 m/sec. Therefore, its cosine is, at best, of the order of $1/3$. (If ξ is compared to unity from which it differs very little in the

expression (150) of x_e , we get $x_e \cos \beta_e = \sqrt{1 + 2(1 - h)Q \cos^2 \beta_e}$ which shows that $x_e \cos \beta_e$ is certainly greater than unity as long as h ranges between 0 and unity.)

The term $c_{r_e} \rho_a / 2 \rho_e x_e \cos \beta_e$ appears, as before, to be of the order of 0.10 and therefore negligible in first approximation before unity.

In a number of particularly interesting cases, this term has a much lower value, and then it is entirely legitimate to disregard it.

To simplify the general discussion of equation (151), the approximation is hereinafter considered legitimate by comparing the approximate value of η_p to the exact value computed by (151).

The approximate formula of the evaluation of η_p is

$$\eta_p = h\eta_h + (1 - h) \frac{2\xi}{\xi x_e^2 - (1 + \xi \tan^2 \beta_e)} \left\{ (1 + \eta_h \tan^2 \beta_e)(x_e \cos \beta_e - 1) + \frac{\xi - 1}{\xi} \right\} \quad (152)$$

In the propulsion system of the general type (fig. 34), the propulsion is achieved together by the propeller thrust and by the helicoidal reaction of the rockets, the axial component of which contributes to the real thrust of the complete system.

In the second member of (152), the term $h\eta_h$ can be regarded as representing the contribution of the propeller itself to the propulsive efficiency η_p . The rest of the second member should then be considered as representing the contribution of the revolving rockets and can therefore be expressed in the form $(1 - h)\eta_f$, η_f designating the propulsive efficiency of the revolving rockets.

By definition

$$\eta_p = h\eta_h + (1 - h)\eta_f \quad (153)$$

and the approximate expression of η_f deduced from (152), where x_e is replaced by its value furnished by (150), becomes

$$\eta_f = \frac{1}{(1-h)Q} \left\{ (1 + \eta_h \tan^2 \beta_e) \left(\sqrt{\xi [1 + 2(1-h)Q \cos^2 \beta_e]} - \xi \right) + \xi - 1 \right\} \quad (154)$$

Before proceeding to the discussion of the efficiencies η_g , η_p , η_f , calculated by the preceding formulas, it will be shown that these formulas comprise, by virtue of particular cases, several formulas already established or utilized in the first part of the present report where the influence of the resistance of the rockets (characterized by c_{re}) had been neglected.

(1) Simple rocket.— In this case, there is no propeller ($h = 0$) and the exhaust is fixed and axial ($\tan \beta_e = 0$). The formulas (153) and (154) give then

$$\eta_p = \eta_f = \frac{1}{Q} \left[\sqrt{\xi(1 + 2Q)} - 1 \right]$$

which is identical with (90), when ξ is assimilated to unity.

(2) Normal engine with ordinary pusher propeller.— The exhaust is still fixed and axial ($\tan \beta_e = 0$), but it involves a propeller which practically absorbs all the effective work of the thermodynamic cycle ($h = 1$). Formulas (153) and (154) then give

$$\eta_p = \eta_h \quad \eta_f = 0$$

Therefore, the formulas established above, namely,

(136) and (137) for the heat balance of the engine \underline{M} and of the rotating system \underline{C}

(145) for the real thrust T

(150) for the corresponding speed of exhaust $w_e = x_e V$

(152) for the propulsive efficiency η_p

(153) and (154) for the approximate expression of η_p and η_f

(8) for the over-all efficiency η_g

can be regarded as forming a general theory of propulsion systems as represented by figure 34.

48. Study of the propulsive efficiency

The discussion involves formulas (153) and (154). The parameters are ξ , h , Q , η_h , and β_e .

(a) The coefficient ξ , for liquid fuel similar to kerosene for example, is, at most, equal to about 16/15¹³ and tends toward unity when the dilution of the mixture increases. For simplification ξ can be compared to unity, and formula (154) replaced by¹³

$$\eta_f = \frac{1 + \eta_h \tan^2 \beta_e}{(1 - h)Q} \left[\sqrt{1 + 2(1 - h)Q \cos^2 \beta_e} - 1 \right] \quad (155)$$

(b) The partition coefficient h can be positive or negative. (The system called "engine M" (fig. 34) functions then as receiver.)

The values $h = 0$ and $h = 1$ are known to correspond to the pure jet propeller and to the classical engine-propeller system respectively. Coefficient h can exceed unity in a certain measure, but is upwardly limited by the value h_1 , for which the quantity below the radical in (150) of x_e cancels out. Therefore

$$h < h_1 = 1 + \frac{1 + \xi \tan^2 \beta_e}{2Q} \quad (156)$$

The value $(\eta_p)_1$ of the propulsive efficiency for $h = h_1$ is approximately (ξ being compared to unity)

$$(\eta_p)_1 = \eta_h - \frac{1}{Q} \left[1 + \frac{\eta_h (\tan^2 \beta_e - 1)}{2} \right] \quad (157)$$

This value of η_p is, in general, positive, that is to say, the system, in fact, propulsion (real positive thrust), because the parameter Q has a fairly high value in the important cases in practice.

¹³On examination of the absolute error introduced in η_p by comparing ξ to unity, it is found that the modulus of this error is, at most, of the order of 1/10 Q and consequently very small after the parameter Q exceeds several unities, which is the general case for the susceptible realizations under consideration. Therefore, the above approximation remains valid except when Q is too small. In that event, the correct formula (154) must be used for computing η_f .

The value h_1 therefore constitutes the effective upper limit of h . When $(\eta_p)_1$ was negative, this upper limit will be lower and reduced to the value of h which cancels η_p and corresponds to a change in sign of the real thrust.

(c) The parameter Q is defined by (148). Consider a fuel similar to kerosene for which L is of the order of 11,000 cal/kg, or, in M.K.S. units, $L = 45,900,000$.

The practical field can be defined¹⁴ by

η_{th} ranging between 0.20 and 0.45

a ranging between 20 and 75

V ranging between 25 and 200 m/sec (90 and 720 km/h)

the parameter Q ranges between the extreme values

$$Q = 3.06 \quad \text{and} \quad Q = 1650$$

The range involved at present or in the near future is defined by the following extreme cases:

$$L = 11,000 \text{ cal/kg} \quad \eta_{th} = 0.30 \quad a = 20 \quad V = 117 \text{ m/sec} = 420 \text{ km/h} \\ \text{for } Q = 50$$

$$L = 11,000 \text{ cal/kg} \quad \eta_{th} = 0.20 \quad a = 20 \quad V = 55.5 \text{ m/sec} = 200 \text{ km/h} \\ \text{for } Q = 150$$

(d) The propeller efficiency η_h should be as high as possible; a range between $\eta_h = 0.60$ and $\eta_h = 0.80$ can be assumed.

(e) The quantity $\tan \beta_e = U_e/V$ ranges between 0 (rockets stationary, jet axial) and an upper limit which corresponds to the maximum of the peripheral speed admissible for a rotating system. The application of propellers affords around 400 m/sec, while that of turbines and turbomachines scarcely exceeds 300 m/sec. An upper limit of 400 m/sec is an optimistic assumption, since the revolving rockets constitute a system that approaches the turbine by reason of its conditions of thermal fatigue. When V ranges between 25 and 200 m/sec, $\tan \beta_e$ is seen not to exceed a limit of between 16 and 2.

¹⁴These values can be referred to those contemplated for the study of the rockets in article 25. The extreme values of η_{th} were reduced here by reason of the difficulty in discounting a thermal efficiency approaching 50 percent (article 32).

The parameters Q and η_h being regarded as fixed (within the limits visualized above) and ξ being compared to unity, the efficiency η_p to be studied is dependent only upon $\tan \beta_e$ and h . The values of η_p according to (153) and (154) are discussed in these conditions.

49. Influence of $\tan \beta_e$ or of the ratio U_e/V

The partial derivative of η_p with respect to β_e has the value

$$\frac{\partial \eta_p}{\partial \beta_e} = \frac{2 \tan \beta_e}{Q \cos^2 \beta_e} \times \left\{ \frac{\eta_h \left[1 + (1 - h)Q \cos^2 \beta_e - \sqrt{1 + 2(1 - h)Q \cos^2 \beta_e} \right] - (1 - h)Q \cos^2 \beta_e}{\sqrt{1 + 2(1 - h)Q \cos^2 \beta_e}} \right\} \quad (158)$$

It is readily seen that this derivative has not a constant sign. It cancels out for the following real or imaginary values of β_e

$$\tan \beta_e = 0 \quad \cos^2 \beta_e = 0 \quad \cos^2 \beta_e = \frac{\eta_h}{1 - \eta_h} \left[1 \pm \sqrt{\frac{2(1 - \eta_h)}{\eta_h(1 - h)Q}} \right]$$

For $\cos \beta_e = 0$ or infinite $\tan \beta_e$, it is certain that η_p passes through a maximum or minimum whose value

$$\eta_p = \eta_f = \eta_h \quad (159)$$

is, moreover, independent of Q and h .

For $\tan \beta_e = 0$, η_p takes the value

$$\eta_p = h\eta_h + \frac{1}{Q} \left[\sqrt{1 + 2(1 - h)Q} - 1 \right] \quad (160)$$

which obviously totalizes the effect of a pusher propeller and of a fixed rocket with radial jet. This value is either a maximum or a minimum, depending upon whether η_h exceeds or falls below the quantity

$$0.5 \left[1 + \frac{1}{\sqrt{1 + 2(1 - h)Q}} \right]$$

Thus, while it is of interest to increase $\tan \beta_e$, the permissible maximum peripheral speed U_e of revolving rockets imposes a limit.

50. Influence of the division factor h

The partial derivative of η_p with respect to h is

$$\frac{\partial \eta_p}{\partial h} = \eta_h - \frac{(1 - \eta_h \tan^2 \beta_e) \cos^2 \beta_e}{\sqrt{1 + 2(1 - h)Q} \cos^2 \beta_e} \quad (161)$$

This derivative cancels out for the unique value h_2 of h given by

$$h_2 = 1 - \frac{(1 - \eta_h)(1 + \eta_h + 2\eta_h \tan^2 \beta_e)}{2Q\eta_h^2(1 + \tan^2 \beta_e)} \quad (162)$$

This value is less than unity. It is therefore certain that (156) is satisfied (that is, that x_e is real) and that the real thrust is positive, that is to say, the system is actually propulsive. For $h = h_2$, the efficiency η_p passes through a maximum greater than η_h and is therefore positive.

The relative difference Δ of this maximum with respect to the propeller efficiency η_h is

$$\Delta = \frac{(\eta_p)_{\max} - \eta_h}{\eta_h} = \left(\frac{1 - \eta_h}{\eta_h} \right)^2 \frac{\cos^2 \beta_e}{2Q} \quad (163)$$

to the extent that η_h is smaller, as $\tan \beta_e$ approaches nearer to zero and Q becomes smaller.

Hence, it can be positively stated that, with η_h and Q considered as fixed and invariable, the arrangement of the jet propeller improves the propulsive efficiency η_p with respect to the usual engine-propeller system whenever this arrangement functions with a coefficient of distribution $h = h_2$ defined by the condition (162).

The limits of this possible improvement for the extreme cases of interest in actual practice in aviation or in the near future are evaluated.

The extreme cases are those defined in article (48) by

$$L = 45,900,000 \quad \eta_{th} = 0.2 \quad a = 75 \quad V = 200 \quad Q = 3.06$$

$$L = 45,900,000 \quad \eta_{th} = 0.45 \quad a = 20 \quad V = 25 \quad Q = 1650$$

For an assumed peripheral speed U_e limited to 400 m/sec, $\tan \beta_e$ ranges between 0 and 2 and 0 and 8, respectively.

On the other hand, η_h may be estimated as ranging between 0.6 and 0.8. Under these conditions, it is possible to set up, utilizing (162) and (163), table XXII¹⁵, indicating the value of h which supplies the corresponding maximum improvement Δ in propulsive efficiency η_p (with respect to the engine-propeller system) as well as the value of Δ and of the corresponding maximum of η_p .

Table XXII

Q	$\tan \beta_e$	η_h	h_2	Δ	$(\eta_p)_{\max}$
3.06	0	0.6	0.709	0.0726	0.6435
		.8	.908	.0102	.8082
	2	.6	.768	.0145	.6065
		.8	.916	.00204	.80163
1650	0	.6	.99946	.000135	.600081
		.8	.99983	.000019	.800015
	8	.6	.99959	.000002	.6000012
		.8	.99985	.0000003	.80000024

¹⁵Table XXII gives the value of h for the case where $\tan \beta_e = 0$. It may seem surprising that we contemplate a value (0.6 or 0.8) for the propeller efficiency η_h and that the latter could contribute to the propulsion, since $U_e = V \tan \beta_e$ is then found zero. Actually, U_e represents only the peripheral speed of the propeller if the latter was obligatorily joined to the arms which carry the exhaust rockets. But this disposition, exemplified in figure 34, is not obligatory. The case $\tan \beta_e = 0$, with $h \neq 0$, can thus be conceived in principle. It corresponds to an exhaust by stationary rockets and a propeller geared to an engine which in form of mechanical power realizes only a fraction h of the effective power of the complete thermodynamic cycle.

The figures in this table show that the maximum of η_p is very near the value of η_h which in the most favorable case ($Q = 3.06$) may not exceed 7 percent, equivalent to a very high propulsion speed of (720 km/h). In this event, this maximum is reached at a value of h near 70 percent, that is, when the engine M transmits to the propeller only 70 percent of the effective power of the complete thermodynamic cycle. For the ordinary cases in practice, as defined by a value Q ranging between 50 and 150, the maximum of η_p is not discernible from the propeller efficiency η_h and is reached when h is practically identical with unity.

51. Variation of η_p in ordinary cases in practice

In order to clarify the preceding algebraic discussion, by numerical examples, there are reproduced in tables XXIII and XXIV the results of the calculation of η_p and its various elements for several values of η_h , $\tan \beta_e$, and h , namely

$$\eta_h = 0.6 \text{ and } 0.8$$

$$\tan \beta_e = 0, 2, 4, 6, \text{ and } \tan \beta_e \text{ infinite}$$

$$h = 0, 0.25, 0.50, 0.75, \text{ and } 1$$

The values of 50 and 150 given in parameter Q appear to comprise the ordinary cases in practice and correspond, for example, to the following conditions:

$$L = 11,000 \text{ cal/kg} \quad \eta_{th} = 0.30 \quad a = 20 \quad V = 420 \text{ km/h} \quad \text{for } Q = 50$$

$$L = 11,000 \text{ cal/kg} \quad \eta_{th} = 0.20 \quad a = 20 \quad V = 200 \text{ km/h} \quad \text{for } Q = 150$$

The data in this table are represented by the curves of figures 35, 36, 37, and 38. On examination of these curves, it is clear that, within the range of application of ($50 < Q < 150$) and when Q and η_h are assumed constant:

(1) η_p increases with $\tan \beta_e$, that is, with the peripheral speed U_e of the rotating rockets when h has a fixed value comprised between zero and unity.

(2) Regardless of the value of $\tan \beta_e$, the maximum of η_p is practically equal to η_h and is obtained for a value h_2 of h practically equal to unity.

(3) It is therefore of interest to have h as near as possible to unity and it is then practically immaterial to make the exhaust rotate at any one speed.

Table XXIII

$$Q = 50$$

		$\eta_h = 0.60$					$\eta_h = 0.80$				
		$h = 0$	$h = 0.25$	$h = 0.50$	$h = 0.75$	$h = 1.0$	$h = 0$	$h = 0.25$	$h = 0.50$	$h = 0.75$	$h = 1.0$
$\tan \beta_e = 0$	$\eta_f =$	0.181	0.206	0.246	0.368	1.000	0.181	0.206	0.246	0.368	1.000
	$(1 - h)\eta_f =$.181	.154	.123	.092	0	.181	.154	.123	.092	0
	$h\eta_h =$	0.	.150	.300	.450	.600	0	.200	.400	.600	.800
	$\eta_p =$.181	.304	.423	.542	.600	.181	.354	.523	.692	.800
$\tan \beta_e = 2$	$\eta_f =$.243	.272	.314	.396	.680	.300	.336	.389	.488	.840
	$(1 - h)\eta_f =$.243	.204	.157	.099	0	.300	.253	.194	.122	0
	$h\eta_h =$	0.	.150	.300	.450	.600	0	.200	.400	.600	.800
	$\eta_p =$.243	.354	.457	.549	.600	.300	.452	.5945	.722	.800
$\tan \beta_e = 4$	$\eta_f =$.344	.376	.418	.484	.625	.447	.491	.542	.628	.815
	$(1 - h)\eta_f =$.344	.282	.209	.121	0	.447	.368	.272	.157	0
	$h\eta_h =$	0	.150	.300	.450	.600	0	.200	.400	.600	.800
	$\eta_p =$.344	.432	.509	.571	.600	.447	.568	.672	.757	.800
$\tan \beta_e = 6$	$\eta_f =$.416	.447	.480	.530	.610	.547	.589	.634	.698	.805
	$(1 - h)\eta_f =$.416	.335	.240	.1325	0	.547	.441	.317	.1745	0
	$h\eta_h =$	0	.150	.300	.450	.600	0	.200	.400	.600	.800
	$\eta_p =$.416	.485	.540	.5825	.600	.547	.641	.717	.7745	.800
$\tan \beta_e = \infty$	$\eta_f =$.600	.600	.600	.600	.600	.800	.800	.800	.800	.800
	$(1 - h)\eta_f =$.600	.450	.300	.150	0	.800	.600	.400	.200	0
	$h\eta_h =$	0	.150	.300	.450	.600	0	.200	.400	.600	.800
	$\eta_p =$.600	.600	.600	.600	.600	.800	.800	.800	.800	.800

Table XXIV

 $Q = 150$

		$\eta_h = 0.60$					$\eta_h = 0.80$				
		$h = 0$	$h = 0.25$	$h = 0.50$	$h = 0.75$	$h = 1.0$	$h = 0$	$h = 0.25$	$h = 0.50$	$h = 0.75$	$h = 1.0$
$\tan \beta_e = 0$	$\eta_f =$	0.109	0.125	0.150	0.206	1.000	0.109	0.125	0.150	0.206	1.000
	$(1 - h)\eta_f =$.109	.093	.075	.051	0	.109	.093	.075	.051	0
	$h\eta_h =$	0	.150	.300	.450	.600	0	.200	.400	.600	.800
	$\eta_p =$.109	.242	.375	.501	.600	.109	.293	.475	.651	.800
$\tan \beta_e = 2$	$\eta_f =$.154	.175	.208	.272	.680	.190	.216	.256	.336	.840
	$(1 - h)\eta_f =$.154	.131	.104	.068	0	.190	.162	.128	.084	0
	$h\eta_h =$	0	.150	.300	.450	.600	0	.200	.400	.600	.800
	$\eta_p =$.154	.281	.404	.518	.600	.190	.362	.528	.684	.800
$\tan \beta_e = 3$	$\eta_f =$.195	.219	.256	.326	.640	.250	.282	.328	.420	.820
	$(1 - h)\eta_f =$.195	.164	.128	.082	0	.250	.2105	.164	.105	0
	$h\eta_h =$	0	.150	.300	.450	.600	0	.200	.400	.600	.800
	$\eta_p =$.195	.314	.428	.532	.600	.250	.4105	.564	.705	.800
$\tan \beta_e = 4$	$\eta_f =$.234	.263	.302	.376	.625	.306	.342	.394	.488	.814
	$(1 - h)\eta_f =$.234	.197	.151	.194	0	.306	.256	.197	.122	0
	$h\eta_h =$	0	.150	.300	.450	.600	0	.200	.400	.600	.800
	$\eta_p =$.234	.347	.451	.544	.600	.306	.456	.597	.722	.800
$\tan \beta_e = 5$	$\eta_f =$.271	.300	.341	.392	.615	.356	.395	.448	.516	.806
	$(1 - h)\eta_f =$.271	.225	.1705	.098	0	.356	.295	.224	.129	0
	$h\eta_h =$	0	.150	.300	.450	.600	0	.200	.400	.600	.800
	$\eta_p =$.271	.375	.4705	.548	.600	.356	.495	.624	.729	.800
$\tan \beta_e = 6$	$\eta_f =$.305	.334	.377	.445	.610	.401	.440	.496	.588	.805
	$(1 - h)\eta_f =$.305	.250	.1885	.1115	0	.401	.330	.248	.147	0
	$h\eta_h =$	0	.150	.300	.450	.600	0	.200	.400	.600	.800
	$\eta_p =$.305	.400	.4885	.5615	.600	.401	.530	.648	.747	.800
$\tan \beta_e = \infty$	$\eta_f =$.600	.600	.600	.600	.600	.800	.800	.800	.800	.600
	$(1 - h)\eta_f =$.600	.450	.300	.150	0	.800	.600	.400	.200	0
	$h\eta_h =$	0	.150	.300	.450	.600	0	.200	.400	.600	.800
	$\eta_p =$.600	.600	.600	.600	.600	.800	.800	.800	.800	.800

In these conditions (Q and η_h constant), it can thus be concluded that the rotating exhaust arrangement is of no importance for the improvement of propulsive efficiency η_p defined by the fundamental formula (9).

52. Study of over-all efficiency η_g

The over-all efficiency η_g being, by definition, the product of the thermal efficiency η_{th} by the propulsive efficiency η_p , it is readily deduced that:

When η_{th} , $Q = \eta_h L/aV^2$, and η_h are fixed and constant, the rotating exhaust system cannot, in the ordinary cases of practice, produce an appreciable improvement in the over-all efficiency η_g .

The assumption of η_{th} constant is justified in the comparison of various propulsion systems embodying a propeller.

To make the preceding conclusion void, it is therefore necessary to envisage the means of modifying η_{th} or Q , or both, for a rotating exhaust system, in such a way that the product $\eta_{th} \times \eta_p = \eta_g$ is increased with respect to the classical engine propeller system. Of course, it is assumed that the system in question operates at the same speed of propulsion and consumes the same fuel. The quantities V and L being thus fixed, it is seen that the desired result is only obtainable by a modification of η_{th} and a .

These quantities being intimately related, the discussion is reduced to the thermal efficiency η_{th} of the various systems which can be visualized, namely, one obtained by adapting a rotating exhaust to a reciprocating engine (with explosion or injection) of the conventional type, or one obtained by special adaptation of an engine of suitable type to the rotating exhaust system.

53. Rotating exhaust adapted to a standard reciprocal engine

In the standard airplane engine, the cylinders empty periodically in the exhaust pipes and the latter evacuates the burnt gases in the atmosphere.

The multiplicity of cylinders and the high frequency of their exhaust phases are the reason that the exhaust pipes deliver a mixture of gas (and superheated water vapor comparable to gas) whose state and speed are practically constant. It is this state, identified by subscript e , that was taken into consideration in the definition (4) of the thermal efficiency η_{th} , which was adopted. The corresponding pressure is equal to that of the surrounding medium and the speed of

ejection w_e is ordinarily so small that its propulsive reaction is insignificant and can be disregarded. If a jet turbine is mounted on this point, practically no useful power is realized without raising again the exhaust pressure in the engine, which involves a reduction of the effective energy.

There is only one means by which the addition of an exhaust turbine to an engine affords a gain in effective energy without modifying the internal cycle and that is by lowering the external pressure which is produced naturally when an airplane ascends in the air. This is the principle also of Rateau's supercharging method, a method by which the gain in effective power thus obtained is utilized to supply the engine with compressed air at standard atmospheric pressure. It affords the reestablishment of the rated horsepower of the engine.

The rotating exhaust system whose adaptation to a standard engine is being investigated operates as a gas turbine of the helical-centrifugal type, according to the well-known principle of the reaction wheel.

By means of this device, the exhaust pressure of the engine can be raised, kept constant, or lowered, when the outside pressure is assumed constant and the question is whether the thermal efficiency η_{th} of the whole thermodynamic cycle can be improved by this method.

However, before dealing with this question, it is advisable to examine closely the (essentially discontinuous) functioning of the exhaust of a standard engine and its effect on the thermal efficiency defined by formula (4).

Consider, by way of illustration, the Watt diagram of a standard four-stroke-cycle engine (fig. 39).

The exhaust phase is represented by the arc gj of the curve traced by the indicator. During the quasi-totality of the corresponding stroke of the piston, the pressure in the cylinder is sensibly constant. This "internal exhaust pressure" is denoted by p_i .

On the outside of the cylinder the evacuated gases, starting from a variable and intermediate state between g and j , arrive at the exhaust pipe exit in a supposedly uniform state (p_e, T_e), and at a uniform speed w_e . It is this outside exhaust state that is used in the definition (4) of the thermal efficiency η_{th} of the total thermodynamic cycle comprised between inlet and exit of the engine.

Assuming that the transformation gj in the cylinder, as well as the flow of the gases in the exhaust pipes is adiabatic, that is, without exchange of heat with the walls, an assumption that is practically realized in the usual engines, and denoting by U and V the internal energy and the volume of the mass M of the active bodies per unit mass of fuel and by ϵ the ratio to this mass of that of the residual gas in the cylinder in state j when the exhaust is terminated, the application of the law of conservation of energy to the residual gases during the adiabatic transformation gj ¹⁶ gives

$$-p_i(V_j - V_g) = U_j - U_g \quad (164)$$

or by comparing these gases to perfect gases and designating the ratio of the specific heats by $\gamma = C/c$ (at constant pressure and constant volume)

$$T_j = T_g \left[\frac{1}{\gamma} + \left(1 - \frac{1}{\gamma} \right) \frac{p_i}{p_g} \right] \quad (165)$$

a relation which gives T_j as function of T_g , when $\frac{p_i}{p_g}$ is fixed.

It is noted that, $\frac{p_i}{p_g}$ being less than unity, T_j is less than T_g .

This formula enables the corresponding mass ϵ of the residual gases to be computed with respect to the evacuated gases. With τ as the ratio of volumetric compression, (maximum volume/minimum volume of cylinder) we get

$$\tau \epsilon V_j = (1 + \epsilon) V_g$$

or, with allowance for (165) and the equation of state of perfect gases $p\sigma = (C - c)T$

$$\epsilon = \frac{1}{(\tau - 1) + \frac{\tau}{\gamma} \left(\frac{p_g}{p_i} - 1 \right)} \quad (166)$$

¹⁶This transformation is, essentially, irreversible by reason of the sudden expansion produced in g at opening of the exhaust. In the present instance the arc gj of the diagram was compared to the two rectangular segments at right angle.

The law of conservation of energy is next applied to the total evacuated and the residual gases, during the total exhaust ge,

$$-p_e V_e + p_i \left[(1 + \epsilon) V_g - \epsilon V_j \right] = U_e + \epsilon U_j - (1 + \epsilon) U_g + \frac{M}{2} w_e^2$$

Equation (164) and the law of perfect gases (particularly the law $\underline{U} + p\underline{V} = MCT + Cte$) make it possible to express the foregoing relation in the form

$$T_e = T_g \frac{1 + (\gamma - 1) \frac{p_i}{p_g}}{\gamma} - \frac{w_e^2}{2C} = T_j - \frac{w_e^2}{2C} \quad (167)$$

The outside exhaust temperature T_e is therefore lower than the residual gas temperature T_j (final internal exhaust temperature) by $w_e^2/2C$, the difference being so much greater as the gas in state e is exhausted at a greater speed.

It will be noted that, since the transformation je is adiabatic and irreversible, the entropy of the gases is greater in e than in j, and, as T_e is lower than T_j , the pressure p_e is certainly lower than p_i . The loss of head ($p_i - p_e$) of the cylinder at the exhaust outlet depends on w_e according to a definite law, but is difficult to specify because the latter is influenced, notably, by the shape of the valves, by their law of opening, as well as by the shape of the exhaust pipes, the number of cylinders, etc.

In any case, the foregoing summary analysis permits the circumstances which determine j and e to be distinguished.

The pressure p_e is given and equal to p_a .

Consider, on the other hand, the state g as being given. The speed w_e at the exhaust outlet can be modified by altering the shape and the sections of the valves and pipes. The loss of head ($p_i - p_e$) depends primarily on w_e and increases with w_e .

Then it is deduced from (167) that, state $g(p_g, T_g)$ being given as well as pressure $p_e = p_a$, an increase in w_e is accompanied by an increase in p_i , T_j , and $^{17}\epsilon$.

¹⁷The rise of T_j and ϵ slightly modifies the filling of the cylinder, during the induction phase as well as the action of the walls and therefore involves a general modification of the thermodynamic cycle which does not leave the state g rigorously constant. But these modifications are altogether of secondary order if w_e is not made to vary abnormally.

As regards the variation experienced by T_e , nothing can be affirmed, a priori, concerning its sign, as the latter depends upon the rise of the back pressure (or internal pressure) p_i of the exhaust, that is, the loss of head ($p_i - p_e$) of which the law, as function of w_e , is not sufficiently known to permit definite conclusions.

It might be asked whether it would be of interest to modify the exhaust of a normal engine by adding a turbine which would realize the kinetic energy of the evacuated gases.

It will be seen that, when w_e is increased, the state g (start of exhaust) and the outside pressure p_e being assumed fixed, the back pressure p_i in the cylinder is raised. This reduces the energy of the cylinder, but the kinetic energy acquired at the exhaust outlet can be utilized in the turbine. The energy thus realized can outweigh the diminution of the energy of the engine, if the back pressure in the cylinder is high enough and the turbine efficiency is adequate.

In particular, if p_i is raised up to p_g , that is, exhaust at constant pressure is realized, the loss of energy in the cylinder is approximately equal to $\frac{\gamma}{\gamma - 1} V_g (p_g - p_e)$, whereas the energy supplied to the turbine (utilized by the latter with its proper efficiency) is that of the adiabatic expansion from p_g to p_e or

$$\frac{\gamma}{\gamma - 1} V_g \left[p_g - p_e \left(\frac{p_g}{p_e} \right)^{1/\gamma} \right]$$

which is considerably greater than the previously evaluated energy loss.

If, in these conditions, the exhaust of a normal engine is regulated, without modifying the induction pressure¹⁸, the engine charge and, consequently, its ratio of effective power to weight is considerably reduced. Moreover, the influence accruing from the residual gases lowers the efficiency and modifies the state g , which voids one of the assumptions of the preceding calculations.

This draw-back is removed by making provision for induction at high pressure and as close as possible to the exhaust pressure, but the pressure p_g at start of the exhaust itself is raised and remains higher

¹⁸However, it would be necessary to delay the opening of the intake so that the cylinder is at low pressure and that the feeding could be effected at normal pressure.

than the back pressure. The only advantage accruing from this arrangement (Rateau's procedure of supercharging by exhaust gas turbine) is the increase of power supplied by a given engine. In fact, this method is never applied on airplane engines at sea level, where, by reason of the limitations imposed on the compression temperatures by the risk of auto-ignition and spontaneous detonation, the rate of compression in the cylinder must be reduced, which involves an appreciably lower efficiency considering the fairly inferior efficiency (of the order of 65 to 75 percent) of the turbine and of the compressor which the latter operates. The method is applied only to engines at altitude where the natural decrease in the outside pressure makes the utilization important.

But in injection engines, on the other hand, the supercharging by exhaust turboblower is practical, even at ground level, in particularly favorable conditions.

In all cases, the back pressure p_i in the cylinder is maintained at some value between p_g and p_e which is controlled by modifying the volume or the opening of the exhaust pipes that lead to the turbine. The rotating exhaust presents an arrangement similar to the Rateau turbine, but in which the pressure p_k upstream from the turbine (pressure necessarily lower than the back pressure p_i in the cylinder) can, within certain limits, be higher or lower than the final exhaust pressure p_e .

To determine the effect of this arrangement on the thermal efficiency η_{th} defined by (4), compare a standard engine with that obtained by adapting a rotating exhaust to the former.

For the standard engine

$$\eta_{th}L = (\underline{U} + p\underline{V})_a - (\underline{U} + p\underline{V})_e - \underline{Q}_R \quad (4)$$

and for the engine with rotating exhaust

$$\eta_{th}'L = (\underline{U} + p\underline{V})_a - (\underline{U} + p\underline{V})_{e'} - \underline{Q}_R'$$

Suppose that the exhaust in both engines is adiabatic (in the cylinder as well as in the pipes to permit the use of the above formulas) and that the rotating exhaust modifies neither the heat loss through the walls of the engine nor the work of the passive resistances nor the state g of the gases in the cylinder at incipient exhaust.

Under these conditions, the usual engines show $\underline{Q}_R' = \underline{Q}_R$ and according to the preceding relation

$$(\eta_{th}' - \eta_{th})L = (\underline{U} + p\underline{V})_e - (\underline{U} + p\underline{V})_{e'} = MC(T_e - T_{e'}) \quad (168)$$

The variation ($\eta_{th}' - \eta_{th}$) can be evaluated by simply determining the modification which the rotating exhaust can produce at the terminal exhaust temperature T_e .

Figure 40 is an entropy diagram of the exhaust gases showing the different states and transformations of these gases during the exhaust.

The exhaust, for the normal engine, is assumed to be regulated in such a way as to give it a zero speed w_e . The back pressure p_i is then very near that of the terminal pressure p_e which is given and equal to p_a . The loss of head ($p_i - p_e$) across the valve and the pipes is, in fact, minimum.

The transformation gj of the gases in the cylinder is adiabatic and irreversible (entropy increasing, temperature T_j given by formula (165)). On the other hand, according to (167), the temperatures T_j and T_e are equal. The states g , j , and e are represented, in these conditions, by the corresponding points in figure 40.

For the engine with rotating exhaust, the state of the burnt gases (back pressure p_i) in the cylinder at the end of the exhaust (residuary gases) is denoted by j ; the state and the speed of these gases in the pipes, at entry in the rotating part, the section supposed to be situated in the axis of the rotating system and identified by a steady state of flow of burnt gases, are denoted by k' (p_k' , T_k' , w_k'); and the final state at the outlet of the rotating system by e' .

It is assumed that the state g itself is not modified. On the other hand, it should be admitted that, for acceptable utilization of the engine, the back pressure p_i' is, at best, equal to p_i .

Lastly, the gases in the rotating system are subjected, between state k' and e' , to a compression assumed adiabatic and which is necessarily irreversible, hence of increasing entropy.

The states g , j' , k' , and e' for these conditions are also shown in figure 40. As to the evaluation of the difference ($T_e - T_e'$) of the ordinates of the figurative points e and e' , it should be noted that, p_i' being at best equal to p_i and the irreversibility of the exhaust in the cylinder and with it the entropy of the gases increasing with decreasing p_i' , the point j' is on the right side of point j over an isobar (p_i') situated below the isobar (p_i).

On the other hand, in proportion, as the speed w_k' of the gases in the center of the rotating system is assumed to be greater, the pressure p_k' is lower than the back pressure p_i' . It may be admitted that, in the same conditions, the variation of the entropy ($S_{k'} - S_{j'}$) increases.

Lastly, the rise in entropy ($S_e' - S_k'$) is greater as the difference ($p_e - p_k'$) is greater (in absolute value) and the speed w_k is higher.

In consequence, the point e' is at the right of point e on the isobar ($p_e = p_a$) and T_e' is greater than T_e and η_{th}' smaller than η_{th} , and the difference is so much greater as the pressure p_i' is lower or speed w_k' is higher.

Hence it must be concluded - provided, of course, that the (very plausible) assumptions made here for the sake of simplification of the discussion are legitimate - that the rotating exhaust involves, necessarily, a decrease in the thermal efficiency η_{th} of the thermodynamic cycle as defined by the formula (4).

A lower limit of this decrease can be obtained by neglecting the differences ($p_i - p_e$) and ($p_i' - p_k'$), by assuming the speed w_k' zero like the speed w_e , in which case these differences are minimum and practically very small, lastly, by disregarding the irreversibility of compression of the gases, from $p_k' = p_i'$ to $p_e' = p_a$, in the rotating system. By (165) and (167), we then get for the normal engine

$$T_e = T_g \left[\frac{1}{\gamma} + \left(1 - \frac{1}{\gamma} \right) \frac{p_a}{p_g} \right]$$

and for the engine with rotating exhaust

$$T_e' = T_k' \left[\frac{p_a}{p_i'} \right]^{\frac{\gamma-1}{\gamma}} = T_g \left[\frac{1}{\gamma} + \left(1 - \frac{1}{\gamma} \right) \frac{p_i'}{p_g} \right] \left[\frac{p_a}{p_i'} \right]^{\frac{\gamma-1}{\gamma}}$$

whence, by (168) and after reduction

$$-[\eta_{th}' - \eta_{th}] = \frac{MCT_g}{\gamma L} \left\{ \left(\frac{p_a}{p_i'} \right)^{\frac{\gamma-1}{\gamma}} - 1 + (\gamma - 1) \frac{p_a}{p_g} \left[\left(\frac{p_i'}{p_a} \right)^{\frac{1}{\gamma}} - 1 \right] \right\} \quad (169)$$

Consider, by way of example, the case of a normal type of explosion engine with (in M.K.S. units)

$$T_g = 273^\circ + 700^\circ = 973^\circ \quad p_g = 4p_a (p_a = 10,000)$$

$$MC = 18,000 \quad L = 45,900,000$$

for which, in the conditions cited above

$$T_e = T_j = 784^\circ = 273^\circ + 511^\circ$$

and for the engine whose rotating exhaust is supposed to function with a back pressure reduced to half ($p_i' = 0.5p_a$)

$$T_k' = T_j' = 752^\circ = 273^\circ + 479^\circ$$

$$T_e' = 899^\circ = 273^\circ + 626^\circ = T_e + 115^\circ$$

The decrease in the thermal efficiency of the complete thermodynamic cycle is

$$\eta_{th} - \eta_{th}' = 0.045$$

and this decrease, already appreciable, constitutes a lower limit (for the chosen value of $p_i' = 0.5p_a$) by reason of the ignored irreversibilities. This example further shows that the temperature of the gases at the outlet of the rotating system can be considerably raised, and this fact reveals, if one deals with an explosion engine whose exhaust is always at high temperature as in the chosen example, certain difficulties in the realization of the rotating system and the behavior of its components.

The conclusion is that, no matter how attractive it may appear, the idea of utilizing the rotation of a propeller with hollow blades (terminated by rockets) to insure, with a reduced back pressure, the expulsion of the exhaust gases of a normal aircraft engine can only be achieved at the price of lower over-all efficiency of the propulsion system. This idea, therefore, does not merit being retained.

Note: (1) It should be noted that the drop in η_{th} due to the rotating exhaust is obtained in spite of an increase in indicated energy (and, consequently, in effective energy) of the engine.

In fact, the lowering of the back pressure in the engine itself increases the area of the diagram and, consequently, the indicated energy. But apart from and in consequence of the engine, the irreversibilities which occur have the effect of raising the terminal exhaust temperature T_e' by causing a reduction in the effective energy C_{eff} of the complete thermodynamic cycle, the energy defined by the fundamental formula (3) and which is not identical with the effective energy of the engine.

(2) Incidentally, it is pointed out that, for the effective energy of the heat engine (power available at crankshaft) to be identical with the effective power C_{eff} , it is sufficient that the exhaust be adiabatic and that the difference in the kinetic energy of the active bodies at entry (state a) and at exit (state e) be zero. It is only in these conditions, almost always reached with the usual engines, that the case $h = 1$, with the definition (138) of the coefficient of division h adopted in article 44, corresponds to an engine whose energy C_m , transmitted to the propeller, represents the total effective energy which this engine develops when the complete thermodynamic cycle is actually realized. In practice, this is the case with the normal engine-propeller combination such as used on airplanes.

(3) Lastly, it should be pointed out that lowering the back pressure in the cylinders, permitted by the rotating exhaust, improves the cylinder charge and reduces the losses due to the presence of residuary gases in the free space. It undoubtedly permits, everything else being equal, the extreme pressure ratio of the compression phase to be raised, and at the same time the temperature T_g in the cylinder at the start of the exhaust to be reduced.

These facts, favorable for a certain improvement of the power by weight and the efficiency of the engine, were disregarded in the preceding discussion. They undoubtedly attenuate to some extent the unfavorable conclusion but do not appear to modify the general result.

54. Engine specially adapted to rotating exhaust

The theoretical heat engine, specifically designed for rotating exhaust, includes an adiabatic and reversible compression of the combustion air (solid or liquid) from p_a to $p_c = \lambda_c p_a$, an adiabatic and complete combustion at constant pressure p_c , and an adiabatic and reversible expansion of burnt gases from p_c to p_a . (The characteristics and the thermal efficiency of this theoretical cycle for various compression ratios λ_c and corresponding excess ξ of air in the fuel mixture were given in table XIII.)

In the realization of this theoretical cycle, the imperfection of the real machines modifies the characteristics and the efficiency and it is very important to determine the most favorable conditions of realization as well as the results which can probably be obtained in practice.

To begin with an extreme case, consider an ideal machine which yields the perfect, particularly advantageous, theoretical cycle defined

by $\alpha = 3$, $\lambda_c = 45$, to which corresponds a thermal efficiency of $\eta_{th} = 0.664$ (cf. last line, table XXII) and a parameter Q defined by

$$Q = \frac{\eta_{th} L}{aV^2} = 0.664 \times \frac{0.664 \times 48,000,000}{4 \times 14.68 \times V^2} = \frac{542,000}{V^2}$$

If V varies from 0 to 300 m/sec (0 to 1080 km/h), Q varies from infinity to 6.2 and it is immediately seen that the maximum η_{th} and η_g is reached when the coefficient h is very near unity, that is, when the principal part of the effective energy of thermodynamic cycle is supplied to the rotating system, in form of mechanical energy, by the engine to which it is linked.

Hence, it appears, a priori, that, the conditions are unfavorable when considering systems in which the rotating system receives, in the form of mechanical energy, from the associated system only a reduced, zero, or even negative part of the effective energy of the thermodynamic cycle, the latter being assumed defined and constant.

This presumption is confirmed by treatment of the specific case of the propeller driven exclusively by reaction, the case defined by $h = 0$ and in which the propeller is moved only by the tangential reaction of its exhaust rockets, and subsequently, the case of a jet propeller associated with an internal combustion turbine under the best conditions.

The study of the first of these problems then leads to the case of the specific efficiency (the ratio of energy actually supplied to the theoretical energy available) and of the internal efficiency of tangential reaction turbines with one wheel, that is to say, of a gas turbine.

55. Propeller driven exclusively by reaction

In this case ($h = 0$) where the associated system supplies no mechanical energy to the propeller, two different methods can be conceived: either by omitting the associated system, in which case the preliminary compression must be assured in the propeller itself, operating as centrifugal compressor, and the combustion effected in the rotating rockets; or by eliminating any mechanical connection between the propeller and the associated system and conceiving the latter in the form of a balanced engine, that is, supplying no effective energy. This balanced engine may, itself, be presented in two principal forms: either realizing the compression, the combustion, and an adequate portion of the terminal expansion in such a way as to balance the resistant energies of the engine by the corresponding expansion energy (case of integral cycle); or else constituted by a normal engine engaging a compressor that supplies the compressed combustion air to the rotating unit (case of divided cycle).

56. First type, propeller isolated

This type of machine, proposed and studied by various inventors, is tied to the ancient idea of the autocompression gas turbine, an idea encountered in various patents or designs of turbines with gas or with internal combustion. (This idea has been exhaustively studied by Nernst.)

Without touching upon the problems relating to the functioning of the combustion chambers disposed over the periphery of the rotating system, it is expedient to observe that the thermal efficiency is chiefly dependent upon the compression ratio and it is necessary to evaluate the one that permits this arrangement to be realized.

In the rotating system, the power P_a expended on the compression, and which changes the state of the air from $a(p_a, T_a)$ to state $b(p_b, T_b)$, is, on the assumption of adiabatic compression as is practically inevitable for a system of this type,

$$P_a = aC(T_b - T_a) = a \frac{U^2 + V^2 - w_b^2}{2} \quad (170)$$

where

a = volume (by mass) of air

V = corresponding speed of air (axial in theory) at entry in rotating system (state a)

U = peripheral speed of system, at the point where the air reaches state b

w_b = corresponding velocity of air, at the same place, in the rotating system

With ρ_a as the specific efficiency of the compressor with respect to the theoretical adiabatic, the ratio of compression $\lambda_b = \frac{p_b}{p_a}$ obtained reads

$$\lambda_b = \left[1 + \frac{\rho_a p_a}{a C T_a} \right]^{\frac{\gamma}{\gamma-1}} = \left[1 + \rho_a \frac{U^2 + V^2 - w_b^2}{2 C T_a} \right]^{\frac{\gamma}{\gamma-1}} \quad (171)$$

By way of example, take the most favorable conditions by assuming $\rho_a = 1$ and $w_b = 0$; for air: $\gamma = 1.4034$; $(\gamma - 1)/\gamma = 0.2875$; $C = 1000$ (in M.K.S. units) (In this instance the values adopted by Rateau, as the most recent, are used.) and $W^2 = U^2 + V^2$. Formula (171) then reads

$$\lambda_b = \left[1 + \frac{W^2}{2000T_a} \right]^{3.48} \quad (172)$$

with which the maximum theoretical compression ratio λ_b obtainable by centrifugal compression can be computed as function of W and of T_a .

By taking the values of T_a corresponding to altitudes 0, 5000 m, and 10,000 m in standard altitude, and varying W from 0 to 500 m/sec, the values λ_b shown in table XXV are obtained.

Table XXV

Theoretical Centrifugal Compression (Adiabatic) of Air
as Function of the Resultant Speed W and

Atmospheric Temperature T_a

$$T_a \begin{cases} T_a = 273^\circ + 15^\circ = 288^\circ & Z = 0 \text{ m} & p_a = p_o = 10,330 \text{ kg/m}^2 \\ T_a = 255^\circ & Z = 5,000 \text{ m} & p_a = 0.534 p_o \\ T_a = 223^\circ & Z = 10,000 \text{ m} & p_a = 0.261 p_o \end{cases}$$

	$W = 0$ (m/sec)	100	200	300	400	500
$T_a = 288^\circ$	$\lambda_b = 1$	1.0627	1.2634	1.6562	2.3482	3.506
$T_a = 255^\circ$	$\lambda_b = 1$	1.0699	1.3000	1.7580	2.5774	4.0013
$T_a = 223^\circ$	$\lambda_b = 1$	1.0801	1.3489	1.8942	2.9081	4.7048

These data are represented by the curves of figure 41, which also contains a diagram by means of which $W = \sqrt{U^2 + V^2}$ can be immediately computed as function of U and of V .

In practice, V cannot exceed 350 to 400 m/sec (figures still not reached up to now), V may be considered as ranging between 0 and 200 m/sec, and the efficiency ρ_a of the adiabatic centrifugal

compressor certainly does not exceed 0.85. On the other hand, the speed w_b cannot be cancelled and must maintain a certain value to assure the volume of the rockets with moderate sections of passage.

When these conditions are taken into account, it is easy to see that a compression ratio λ_b of the order of 3 for a single stage of compression is practically the limit.

Consequently, the thermal efficiency of the system is limited to a value, which, theoretically and according to formula (91) where $\gamma = 1.35$, cannot exceed 0.245; and the inevitable irreversibilities of the realization reduce this figure even more (by one-fourth at least).

The system of single propeller actuated exclusively by reaction is condemned by this fundamental defect.

57. Second type of propeller actuated solely by reaction: driving system attached and independent

A. Integral cycle. - In this case the active bodies go through the same cycle and effect their passage across the machine along the same path.

The cycle is represented on a diagram (p, V), or Clapeyron diagram (figure 42).

In the ideal or theoretical machine, this cycle comprises: the irreversible and adiabatic compression ab , from p_a to p_b ; the adiabatic and complete combustion bc , at constant pressure $p_c = p_b$; the reversible adiabatic expansion ce , from p_c to $p_e = p_a$. In the so-called motive system is effected the part $abcd$ of the total cycle, the expansion of which is arrested at pressure p_d so that the effective energy of this system is zero. The rest of the cycle, that is, the complement (de) of the expansion, is effected in the rotating system which does not receive the mechanical energy of the attached system and consequently permits of no mechanical connection with the latter.

In the real machine, the cycle differs from the theoretical cycle $abcde$ by reason of irreversibilities of every nature (effect of walls, residual gases, friction, leakages, etc.).

Now it is proposed to calculate the thermal efficiency η_{th} of the real cycle in probable or admissible conditions.

Two principal forms of realization can be conceived, depending on whether the motive or attached system which produces $abcd$, is of the

reciprocating type (balanced pistons engine) or of the rotary type (balanced internal combustion turbine). In any event, the complement of the cycle is realized in the gas turbine which constitutes the rotating system.

(a) Case where the motive system is of the reciprocating type. - Incidental to the calculations summarized in table XIV, it is stated (by neglecting the kinetic energy of the air at its entrance in the motive system as in the cited calculations - this approximation is admissible here as long as the speed of propulsion V does not exceed 150 to 200 m/sec) that at compression ratios $\lambda_c = \lambda_b = \frac{p_b}{p_a}$ of the order of 15 to 45 the real exhaust temperature T_d' (at constant pressure $p_d' = \lambda_d' p_a$) of the reciprocating engine shifts from 2235° to about 975° when the dilution α passes from 0 to 3, that is to say, when the corresponding excess of air in the combustible mixture increases from 0 to 300 percent.

On leaving the engine, the gases circulate in the pipes which convey them to the rotating system. Since all these parts are practically at a steady state, it is imperative that the temperature of the gases be not excessive¹⁹ and, on considering a temperature of 700°C as practical limit, it is seen that a dilution α of at least equal to 3 must be visualized.

This is the value chosen. With \bar{C}_d'' (cf. table XIV) as the theoretical energy of the reversible expansion beginning with the conditions (p_d', T_d') of the exhaust of the real engine and ρ_d'' as the efficiency of expansion with respect to the reversible adiabatic, the thermal efficiency η_{th} defined by formula (4), is expressed by the formula

$$\eta_{th} = \frac{\rho_d'' \bar{C}_d''}{L} \quad (173)$$

The efficiency of expansion ρ_d'' is not identical with the specific expansion of the exhaust pipe system + rotating system, considered as a turbine. It differs from it by the fact that the kinetic energy remaining in it is excluded from the losses. Therefore, a fairly high value for ρ_d'' , such as $\rho_d'' = 0.90$ for example, can be chosen.

¹⁹ This reservation is not formulated for the liquid-fuel rocket because, for its expansion nozzle, a realization assuring a good behavior of the nozzle (or its convergent part which is very short) can be envisaged in spite of temperatures above 700°C over a little extended zone.

In these conditions the efficiency η_{th} assumes the values indicated in table XXVI.

Table XXVI

Mixture very diluted: $\alpha = 3$ rotating system: $\rho_d'' = 0.90$

Losses in the engine computed as in table XIV ($l = 0.10$)

$\lambda_c = \frac{P_c}{P_a}$	T_b' (deg)	T_c' (deg)	\bar{C}_a' (cal)	T_d' (deg)	$\lambda_d' = \frac{P_d'}{P_a'}$	\bar{C}_d'' (cal)	$\eta_{th} = \frac{0.9\bar{C}_d''}{L}$
15	661	1314	5508	971	3.85	4750	0.372
30	803	1427	7340	974	5.32	5385	.421
45	903	1514	8808	973	6.015	5732	.449

According to this table, the efficiency η_{th} at a compression ratio λ_c ranging between 15 and 45 ranges between 0.372 and 0.449, values which are considerably higher than the thermal efficiency of the ordinary airplane engine (0.27 approximately) and even of the best heavy-oil engines (0.36 approximately).

It is noted that, on comparing the above η_{th} values corresponding to real machines with the corresponding values for the theoretical or ideal machine given by table XIII, namely, 0.517, 0.611, and 0.664, the specific efficiency of the real machine, that is, the ratio $(\eta_{th})_{real}/(\eta_{th})_{theoretical}$, ranges between 0.72 and 0.675, which values are plausible and entirely admissible.

The envisaged type of machine appears therefore distinctly interesting. Its practical realization, however, raises certain difficulties, which are briefly outlined.

When contemplating the realization of the cycle $abcd$ (fig. 42) in the same cylinder, allowance must be made for the free space which increases the losses (by residual gases) and diminishes the volume of the active bodies consumed by the piston stroke. Besides, the volumes in a and d are not equal and the piston must have unequal strokes (the engine operating evidently at four cycles) or else the intake closes (or the exhaust opens) before the piston reaches the bottom of the stroke.

To avoid these drawbacks, this cycle can be contemplated to realize in two twin cylinders operating at two cycles, with transference at the end of compression and with the free spaces restricted to a minimum. This arrangement presents obvious advantages, but at combustion the cylinder operates permanently at high temperatures, although reduced by the dilution $\alpha = 3$ adopted above.

Notwithstanding these objections or reservations, this type of machine merits to be retained.

(b) Case where the motive system is of the rotary type (turbo-machine). - Here a high degree of dilution is evidently indispensable, as for all turbomotors with internal combustion whose real operation is supposed adiabatic as completely and perfectly as possible, along with a moderate compression ratio λ_c in order to lower the temperatures on the blades of the turbocompressor and of the expansion turbine.

As in the study of the turborocket, article 30, a dilution $\alpha = 3$ and a compression ratio λ_c of the order of 10 to 15 is adopted; For the turbocompressor efficiency ρ_a , an extreme value ranging between 0.7 and 0.9 is contemplated; for the efficiency of utilization r of the (primary) expansion turbine which actuates the turbocompressor, a value of the order of 0.75 to 0.85 is assumed. Lastly, the efficiency ρ_d'' (the remaining kinetic energy not counted as loss) of the secondary expansion turbine in the rotating system and in its pipes is allowed for at 0.90.

On these premises, the thermal efficiency η_{th} of the complete thermal cycle reaches the values given in table XXVII and which are to be compared to those of table XVI (with $\rho_d'' = 0.95$ instead of $\rho_d = 0.90$).

Table XXVII

$$(\alpha = 3 \quad T_a = 288^\circ \quad \rho_d'' = 0.90)$$

λ_c	ρ_a	r	η_{th}
10	0.9	0.85	0.352
		.75	.302
		.85	.323
	.8	.75	.268
		.85	.285
		.75	.219
15	.9	.85	.375
		.75	.310
		.85	.340
	.8	.75	.267
		.85	.298
		.75	.215

The continuously progressing turbomachine technique does not permit immediately to envisage more favorable conditions than

$$\lambda_c = 10 \quad \rho_a = 0.8 \quad r = 0.8$$

or

$$\lambda_c = 15 \quad \rho_a = 0.75 \quad r = 0.75$$

In these conditions, the preceding table indicates that the efficiency η_{th} does not exceed 0.28 to 0.23 and consequently will be less than that of good internal combustion aircraft engines and much below that of fuel-injection engines.

Hence the type of machine visualized is predicated upon the premise of considerable progress being made first on the specific efficiency (ρ_a and r) in the utilized turbomachines.

58. Case of divided cycle

The motive system is joined to the rotating system which produces no effective energy, by means of a normal engine which drives an air compressor designed to feed the rotating system. In this arrangement the combustion of the mixture intended for the rotating system is no longer effected in the attached system.

It can be secured either in fixed combustion chambers located between the two systems or in the rotating system itself and then preferably in chambers disposed over its periphery.

The essential difference between these two conceptions is that, in the second system, the arms (or links) of the rotating system cannot be utilized as centrifugal compressors and so permit increasing the compression ratio of the air which arises from the compressor actuated by the auxiliary system.

As long as the speeds V and U are not excessive, the energy of compression of the rotating system remains small compared to that demanded of the attached system, it being known that a total compression ratio λ_c of 10 to 15 must be reached to assure a satisfactory cycle. On the other hand, the difficulties of installation and operation of the compression chambers rotating at a high speed U make the first method preferable, in theory, and which is therefore considered here and its efficiency η_{th} evaluated.

\underline{C}_a = theoretical energy of compression of the air destined to burn 1 kg of fuel in the principal system, the compression effected according to a ratio λ_c determined by the extreme pressure $\lambda_c = p_c/p_a$

ρ_a = specific efficiency (with respect to the theoretical adiabatic) of the compressor driven by the auxiliary system

\underline{C}_d = energy of the theoretical expansion from p_c to p_a , starting from the state in which the gases are introduced, after real compression, by complete and adiabatic compression (to which the real combustion, sufficiently exact in the case in point, is assumed comparable)

ρ_t = efficiency of the real expansion which is effected starting from the fixed combustion chambers and completed at the outlet of the rotating system

$\underline{C}_d' = \rho_t \underline{C}_d$, the thermodynamic energy of the real expansion

η_m = thermal efficiency of auxiliary system assumed to consume the same fuel as the principal system

Lastly, $W_1 = \frac{aV^2}{2g}$ = the initial kinetic energy of the mass of air considered (a = ratio of mass of air/fuel).

The compressor efficiency ρ_a being assumed evaluated under actual operation conditions at speed V , the effective energy \underline{C}_a' absorbed by the compressor is

$$\underline{C}_a' = \frac{\underline{C}_a - W_1}{\rho_a}$$

and necessitates the consumption by the auxiliary system of a quantity of fuel (in kg) equal to $\underline{C}_a'/\eta_m L$ which is additive to the fuel consumed in the same time interval by the principal system.

The effective energy of the complete thermodynamic cycle is $\underline{C}_d' = \rho_t \underline{C}_d$ (3); therefore, to obtain the thermal efficiency η_{th} of the thermodynamic cycle under consideration it is sufficient to refer this energy to the heat value of the fuel consumed $(1 + \underline{C}_a'/\eta_m L)$.

Hence

$$\eta_{th} = \frac{\eta_m \rho_a \rho_t \underline{C}_d}{\underline{C}_a - a \frac{V^2}{2g} + \eta_m \rho_a L} \quad (174)$$

which can be reconciled with formula (97).

Allowance for the initial kinetic energy of the air supplying the compressor of the principal system introduces in the denominator of η_{th} the term $\left(-a \frac{V^2}{2}\right)$ which brings out the beneficial effect of the increase in speed V . It further will be recalled that the specific efficiency ρ_a of the compressor itself depends (but in a rather small measure, especially when a well-adapted turbocompressor is involved) on the speed V at entrance in the orifice of the compressor.

Moreover, in the foregoing expression, the efficiency ρ_t (called the efficiency of the real expansion) represents (if desired) the specific efficiency of the expansion turbine formed by the rotating system, provided that the remaining kinetic energy of the gases at the turbine exit are not included in the losses of the latter.

Assuming that this turbine has a single-rotor playing the part of a rotating distributor and, that, consequently, the losses due to shock, eddies, and friction are reduced correspondingly, the preceding remark leads to envisage a rather high value for the ρ_t in question. Granted that this value could range between 0.90 and 0.95 the lower limit $\rho_t = 0.90$ is systematically adopted to compensate for the effect of the various losses of the complete system (notably the loss through incomplete combustion and adiabatic deficiency of the system).

For the efficiency ρ_a , the values therefore range between 0.7 and 0.9, which holds for good compressors whether of the piston or the rotary type. For the compression ratio λ_c , the uniform value of $\lambda_c = 10$ is assumed, with a dilution $\alpha = 3$ and standard initial temperature $T_a = 288^\circ$. For the thermal efficiency η_m of the auxiliary system, the constant value of $\eta_m = 0.27$ which corresponds to the better modern aircraft engines is adopted.

In these conditions and on the basis of the values of \underline{C}_a and \underline{C}_d given in table XV, formula (173) permits η_{th} to be computed for different values of ρ_a and of speed V .

The data obtained are represented in table XXVIII and plotted in figure 43.

Table XXVIII

$$(\lambda_c = 10 \quad \alpha = 3 \quad T_a = 288^\circ)$$

$$\eta_m = 0.27 \quad \rho_t = 0.9)$$

ρ_a	V (m/sec)	η_{th}
0.9	0	0.316
	150	.323
	300	.350
.8	0	.302
	150	.310
	300	.335
.7	0	.287
	150	.295
	300	.322

Note the appreciable improvement in η_{th} (ρ_t and ρ_a being fixed) when the speed V becomes very high. At speeds below 150 m/sec, this improvement is altogether secondary.

The decrease in η_{th} at decreasing ρ_a is rather moderate and that takes account of the fact that the efficiency of expansion ρ_t in the rotating system, an efficiency which is assumed constant, has a rather high value ($\rho_t = 0.90$).

For a piston compressor, ρ_a values up to 0.90 can perhaps be visualized. For rotary compressors, however, it is prudent not to discount ρ_a values higher than 0.75 at a compression ratio of the order of 10 as above.

Finally, it is seen that, for the systems involved (propellers driven exclusively by reaction and with a compressor geared to auxiliary system), the efficiency η_{th} of the complete thermodynamic cycle could practically reach values of between 0.295 and 0.320, the mixture utilized by the propeller permitting a corresponding excess of air of 300 percent ($\alpha = 3$).

59. Recapitulation

The study, developed in the preceding three paragraphs (55 to 58), of the several visualized propeller systems driven solely by reaction is summarized in the following table XXIX.

Table XXIX

Type of system			Value η_{th} practically obtainable	Compression ratio λ_c visualized
Type I: Propeller isolated			Unsatisfactory because of insufficient compression	Unsatisfactory
Type II: Propeller with attached system	A Integral cycle	a. Balanced engine Reciprocating type	$0.37 < \eta_{th} < 0.45$	$15 < \lambda_c < 45$
		b. Balanced turbomotor	$0.23 < \eta_{th} < 0.28$	$10 < \lambda_c < 15$
	B Divided cycle	Compressor driven by auxiliary engine of standard type	$0.295 < \eta_{th} < 0.32$	$\lambda_c \approx 10$

The limits indicated correspond to satisfactory systems (engines, compressors, turbines) such as those already in existence or apparently easily obtainable. In all of them the dilution of the fuel mixture was fixed at $\alpha = 3$, that is, a corresponding air consumption of $a = 58.8$ for the fuel in question (Rey kerosene).

The foregoing table shows the superiority of the systems of the type (A/a), although certain manufacturing difficulties may be encountered (cf. article 57/a), as well as the weight and bulk which might result, for a reciprocating machine of this type, as a result of the large excess of air ($\alpha = 3$) necessary for the mixture in the cycle.

It is known, from the general discussion in article 51, that a high propulsive efficiency η_p is obtainable with a propeller driven solely by reaction and characterized by $h = 0$. This is very clearly shown in figures 35 and 38.

Nevertheless, the high values of η_{th} , obtainable according to the above table, favor the over-all efficiency $\eta_g = \eta_{th} \times \eta_p$, which constitutes the criterion of appraisal of propulsion systems.

The most favorable (A/a) system is discussed on a model problem.

60. Special study of the several efficiencies of the propeller driven exclusively by reaction, in the most favorable case

The most favorable (A/a) system is characterized by

$$\lambda_c = 45 \quad a = 58.8 \quad \eta_{th} = 0.45$$

The heat value L of the (Rey) kerosene is (in M.K.S. units)

$$L = 48,000,000$$

hence

$$Q = \frac{\eta_{th} L}{a V^2} = \frac{367,000}{V^2}$$

On admitting that V does not exceed 300 m/sec (1080 km/h), the parameter Q remains above 4.08, and, as is easy to verify, the quantity $\xi = \frac{a+1}{a} = \frac{59.8}{58.8} = 1.017$ can be compared to unity for the calculation of the several efficiencies.

In these conditions (with $h = 0$), which characterizes the jet propeller, taken into account, the formulas (153) and (154) give

$$\eta_p = \eta_f = \frac{1}{Q} \left(1 + \eta_h \tan^2 \beta_e \right) \left[\sqrt{1 + 2Q \cos^2 \beta_e} - 1 \right] \quad (175)$$

with $\tan \beta_e = U_e/V$, U_e denoting the peripheral speed of the exhaust sections in the atmosphere.

On the other hand, the over-all efficiency η_g , is by definition

$$\eta_g = \eta_{th} \times \eta_p$$

Quantity Q depends only on V and, by formula (175), η_p depends on Q (or V), U_e , and η_h , the propeller efficiency.

For the latter, a constant value of 0.75 is assumed; U_e is varied from 0 to 400 m/sec, and V from 0 to 300 m/sec.

The results are shown in table XXX and plotted in figure 44.

Table XXX

Values of η_p and of η_g ($\eta_{th} = 0.45$; $\eta_h = 0.75$)

	U_e (m/sec)					
	0	50	100	200	300	400
$V = 100$ (m/sec) $\left\{ \begin{array}{l} \eta_p = \\ \eta_g = \end{array} \right.$	0.207 .093	0.218 .098	0.245 .110	0.323 .145	0.398 .179	0.465 .209
$V = 200$ $\left\{ \begin{array}{l} \eta_p = \\ \eta_g = \end{array} \right.$.370 .167	.374 .168	.3825 .172	.4175 .188	.463 .208	.507 .228
$V = 300$ $\left\{ \begin{array}{l} \eta_p = \\ \eta_g = \end{array} \right.$.497 .224	.500 .225	.503 .226	.5175 .2325	.536 .241	.562 .252

Figure 44 also shows the propulsive efficiency ($\eta_p = \eta_h$) and the over-all efficiency ($\eta_g = \eta_h \times \eta_{th}$) of a normal engine-propeller combination utilizing the same propeller ($\eta_h = 0.75$) and permitting an internal combustion engine, or a heavy-oil injection engine, that is, power plants with an anticipated thermal efficiency equal to 0.27 or 0.34, respectively. A constant value $\eta_h = 0.75$ was adopted for the particular propeller efficiency irrespective of U_e and V , which assumes the said propeller adapted for each case in the best conditions. Besides, it will be more correct to concede a certain decrease in η_h when the

tip speed $(U_e^2 + V^2)^{1/2}$ becomes very high, that is, higher than 300 m/sec. This known decrease in propeller efficiency on approaching sonic velocity was disregarded here for reasons of simplicity.

The diagram of figure 44 proves that within the present practical range of $U_e = 300$ and $V = 200$ m/sec:

(1) The propulsive efficiency η_p of the jet propeller is seen to be considerably below the efficiency η_h of the regular propeller.

(2) The propulsive efficiency η_p increases with the speed V , the peripheral speed U_e being assumed constant.

(3) η_p increases rather slowly with U_e , V being fixed, and passes through a minimum for $U_e = 0$.

(4) The over-all efficiency η_g of the jet propeller reaches, at extreme values of U_e and V , the value corresponding to a normal engine - propeller system.

The conclusion is that, as long as V does not exceed 200 m/sec, and U_e 300 m/sec, the contemplated system remains inferior to a standard system of propulsion in spite of its high η_{th} and its high dilution ($\alpha = 3$), at least for speeds below 200 m/sec.

61. Notes on the internal efficiency of the gas turbine

Figure 44 illustrates the relatively slow increase of η_p with U_e , for given V . This fact calls for an explanation because it may be surprising at first glance. It is, as will be shown, intimately related with the fact that the internal efficiency of the gas turbine does not attain high values except at high rotative speeds.

In the case of the propeller driven exclusively by reaction, the rotating system can be regarded as forming a gas turbine of the centrifugal type and with helicoidal ejection, which drives the propeller the blades of which envelop, in reality, the moving channels of the said turbine.

According to this point of view, the effective horsepower of this turbine on its shaft balances the power absorbed by the propeller. The thrust horsepower consists of the propeller thrust T (fig. 45) and of the axial component T_f of the reaction R_f supported by the turbine and due to the ejection of the gases. These two portions can be

distinguished in the propulsive efficiency η_p^{20} by bringing out the role played by the internal efficiency of the gas turbine formed by the rotating system.

Before presenting η_p in this form, it is well to remember how the internal efficiency of the gas turbine is defined and evaluated by the conventional turbine theory.

Consider a gas turbine, with a single rotor, with central admission and peripheral exhaust, and for greater generalization, admit that this system (fig. 45) is actuated by an axial translation at speed V , so that the resultant W_e of V and U_e , inclined at β_e with respect to the axis of rotation, is opposite to the corresponding speed w_e (with respect to the rotor) of the gases at their discharge.

Let M be the volume by mass of the turbine, d the initial state ($p_d, T_d, w_d = 0$), and e the final state (p_e, T_e, w_e) of the gases passing through the turbine, and \underline{U} and \underline{V} the internal energy and volume of these gases referred to unit mass.

The adiabatic expansion, reversible and very slow, effected from p_d to p_e , conveys the gases from state d to a certain state $E(p_E, T_E)$.

As usual, we put

$$(\underline{U} + p\underline{V})_d - (\underline{U} + p\underline{V})_E = \frac{v_0^2}{2} = gH \quad (176)$$

The thus-defined quantities H and v_0 represent what is called the "head" and the "theoretical velocity" due to this head, of the turbine in question.

²⁰In the case of the true jet propeller ($h = 0$) considered here, the efficiency η_p given by the general formula (175) is reduced to the term η_f called the propulsive efficiency of rotating rockets. It is purposed to recall, that, acting directly through the component T_f of its reaction or, indirectly, through its peripheral component which the propeller transforms into thrust, the jets are the sole motors of the propulsive system.

In the actual expansion, from state d to state e , numerous irreversibilities intervene, as well as more or less important exchanges of heat with the outside.

Let Q_{Rt} be the total heat transferred by the turbine in question and per unit of consumed mass, on the outside; this quantity Q_{Rt} comprises, in particular, the heat of mechanical friction, the bearings being incorporated in the turbine.

Quantity

$$\underline{C} = (\underline{U} + p\underline{V})_d - (\underline{U} + p\underline{V})_e - Q_{Rt} \quad (177)$$

may be called the "effective energy of expansion" (de).

The ratio of effective to theoretical energy, denoted by ρ_d , that is

$$\rho_d = \frac{2\underline{C}}{v_0^2} = \frac{(\underline{U} + p\underline{V})_d - (\underline{U} + p\underline{V})_e - Q_{Rt}}{(\underline{U} + p\underline{V})_e - (\underline{U} + p\underline{V})_E} \quad (178)$$

is conveniently called the efficiency of the real expansion (denoted by ρ_d in formula (173)), but should not be confused with the "internal" or the "specific" efficiency of the turbine.

Quantity P_{eff} is the effective horsepower at the transmission shaft of the turbine operating at head H , that is, with the theoretical horsepower ($Mg H$).

The specific efficiency ρ_t of the real turbine is, by definition, the ratio of the real to the theoretical horsepower or

$$\rho_t = \frac{P_{eff}}{Mg H} = \frac{2P_{eff}}{M v_0^2} \quad (179)$$

The internal horsepower P_1 of the turbine being its effective power augmented by external losses (by external friction and leakage) symbolized by the term $\bar{w}_e MgH$, the internal efficiency ρ_1 , or as it is sometimes called, hydraulic efficiency, is the ratio of internal to theoretical horsepower, that is

$$\rho_1 = \frac{P_1}{Mg H} = \frac{P_{eff}}{Mg H} + \bar{w}_e = \rho_t + \bar{w}_e \quad (180)$$

On the other hand, on a turbine with a single rotor, ρ_1 is expressed by

$$\rho_1 = \frac{2}{v_0^2} (au - a'u') \quad (181)$$

u and u' denoting the peripheral speeds of the moving rotor at the mean radius of the sections of inlet and discharge, a and a' the projections in the peripheral direction of the absolute speed of the fluid at inlet and discharge.

In the case of the gas turbine with central admission and sensibly axial flow (a and u always zero or quasi-zero in every section of the inlet) and tangential and oblique discharge (fig. 45), formula (181) gives

$$\rho_1 = \frac{2U_e(w_e \sin \beta_e - U_e)}{v_0^2} \quad (182)$$

In this study, the loss at the joint (i.e., at the inlet in the rotating system) as well as the losses due to the mechanical friction of the shaft in its bearings are neglected²¹. Accordingly, when it is convenient to reckon the horsepower absorbed by the friction of the rotor in the ambient fluid as effective horsepower, the external losses disappear, the specific efficiency ρ_t is coincident with the internal efficiency ρ_1 according to (18), and by (179) and (182) we get

$$P_{\text{eff}} = \rho_t M \frac{v_0^2}{2} = \rho_1 M \frac{v_0^2}{2} = M U_e (w_e \sin \beta_e - U_e) \quad (183)$$

On the other hand, the application of the principle of the conservation of energy to the turbine and to the elements which it contains, during the time unit and in ratio to the axes in absolute translation with the fixed part of the turbine, yields the relation

²¹ The escape of the active bodies has been consistently disregarded: It is of little significance in good heat machines and can be allowed for by slightly underestimating the thermal efficiency in systematic manner, when its probable value is evaluated.

The bearing friction had not been considered in the rotating system of figure 34, the said system being considered as cantilever construction and its bearings as belonging to the attached or motive system.

$$M[(\underline{U} + p\underline{V})_d - (\underline{U} + p\underline{V})_e - \underline{Q}_{rt}] = \underline{P}_{eff} + M \frac{w_e^2 + U_e^2 - 2w_e U_e \sin \beta_e}{2}$$

or by definition (178) of the real expansion efficiency ρ_d

$$\underline{P}_{eff} = M \rho_d \frac{v_0^2}{2} - M \frac{w_e^2 + U_e^2 - 2w_e U_e \sin \beta_e}{2} \quad (184)$$

Consider the head H (i.e., the speed v_0) as given along with the efficiency ρ_d (which depends only on the changes of state of the active fluids and their thermal exchange with the outside) and put

$V/v_0 = \psi$ = speed of the turbine in axial translation

$U_e/v_0 = \varphi$ = rate of rotation

$w_e/v_0 = \chi$

By (183) and (184)

$$\chi = \sqrt{\rho_d + \varphi^2} \quad (185)$$

a relation which shows that w_e increases indefinitely with U_e .

Entering this value of χ in (182) gives

$$\rho_1 = 2\varphi^2 \left[\sqrt{\frac{\rho_d + \varphi^2}{\psi^2 + \varphi^2}} - 1 \right] \quad (186)$$

which expresses the internal efficiency ρ_1 of the gas turbine (without joint loss) as a function of ρ_d , ψ , and φ .

For a gas turbine at a fixed point (turbine fixed, $\psi = 0$), this expression is reduced to

$$\rho_1 = 2\varphi \left[\sqrt{\rho_d + \varphi^2} - \varphi \right] \quad (187)$$

In the general case, formula (186) shows that ρ_1 tends toward $(\rho_d - \psi^2)$ when φ increases indefinitely.

It is easily checked that ρ_1 increases constantly with φ .

Besides, table XXXI gives the value of ρ_1 for different values of ψ and ϕ , when ρ_d is successively assumed equal to 0.9 or 0.8.

Table XXXI

Internal Efficiency ρ_1 of the Gas Turbine

ρ_d	ψ	$\phi = 0$	0.5	1	2	4	∞
0.9	0	$\rho_1 = 0$	0.572	0.758	0.852	0.880	0.9
	.2	0	.495	.702	.800	.842	.86
	.4	0	.338	.560	.680	.730	.74
.8	0	0	.525	.684	.764	.776	.8
	.2	0	.451	.632	.720	.740	.76
	.4	0	.300	.492	.584	.635	.64

The results of this table are shown plotted in figure 46²². They indicate very plainly the necessity for high speeds of rotation of the order of 1 at least, in order to obtain a beneficial efficiency ρ_1 , especially if ψ is appreciable or ρ_d is low.

The data given for the gas turbine are readily applicable to the true jet propeller described above.

For this system, if the rotating unit is considered as a gas turbine of the impeller type (in translation, at normal operating speed $\psi = \phi / \tan \beta_e$), the effective horsepower (this includes the power absorbed by the friction of the rotor in the ambient fluid, that is, the friction of the propeller blades in the air) of the turbine represents the power P_h absorbed by the propeller, which gives an effective power of propulsion

$$(P_u)_h = T_h V = \eta_h P_{eff} = \eta_h \rho_1 M \frac{v_0^2}{2} \quad (188)$$

On the other hand, the axial component T_f (see fig. 45) of the reaction R_f supported by the rockets supplies an effective power of propulsion

²²These curves have not been extended as far as the origin, where they fit tangentially to the axis of abscissas, in order to avoid the confusion of the curves in this zone resulting from the much-reduced design scale.

$$(P_u)_f = T_f V = \frac{Q_f U_e}{\tan^2 \beta_c} = \frac{P_{eff}}{\tan^2 \beta_c} = \rho_1 \frac{\psi^2}{\phi^2} M \frac{v_0^2}{2} \quad (189)$$

The mass volume M of the turbine (m = volume of fuel, a = corresponding volume of air) is

$$M = m(1 + a)$$

The total effective power of propulsion TV is, by (188) and (189)

$$TV = T_h V + T_f V = \left(\rho_1 \eta_h + \rho_1 \frac{\psi^2}{\phi^2} \right) - m(1 + a) \frac{v_0^2}{2}$$

and the propulsive efficiency η_p by definition (9) is

$$\eta_p = \frac{TV}{m \eta_{th} L} = \frac{(1 + a) v_0^2}{2 \eta_{th} L} \left(\rho_1 \eta_h + \rho_1 \frac{\psi^2}{\phi^2} \right) \quad (190)$$

Quantity η_{th} can be connected with v_0^2 by noting that, by definition of η_{th} :

$$\eta_{th} L = (\underline{U} + p\underline{V})_a - (\underline{U} + p\underline{V})_e - \underline{Q}_R \quad (3)$$

(the mass of combustible mixture including the unit of fuel mass being taken for reference term) and by (178) gives

$$(\underline{U} + p\underline{V})_d - (\underline{U} + p\underline{V})_e - \underline{Q}_{Rt} = \rho_d (1 + a) \frac{v_0^2}{2} \quad (191)$$

lastly, by applying the principle of the conservation of energy between the states a and d to the balanced system attached to the rotating system

$$(\underline{U} + p\underline{V})_a - (\underline{U} + p\underline{V})_d - \underline{Q}_{Rm} = -a \frac{v^2}{2} \quad (192)$$

Adding (191) and (192), by putting $\underline{Q}_{Rm} + \underline{Q}_{Rt} = \underline{Q}_R$ and comparing with (3) gives after reduction

$$(1 + a) \frac{v_0^2}{2 \eta_{th} L} = \left(1 + \frac{a v^2}{2 \eta_{th} L} \right) \frac{1}{\rho_d} \quad (193)$$

Given V , η_{th} , and ρ_d , this relation is used to compute V_0 . Applying this expression to (190) gives the propulsive efficiency in the desired form

$$\eta_p = \frac{1}{\rho_d} \left(1 + \frac{1}{2Q} \right) \left(\rho_1 \eta_h + \rho_1 \frac{\psi^2}{\phi^2} \right) \quad (194)$$

which is none other than formula (175)²³.

By way of illustration, take the system studied in the preceding article (table XXX and fig. 44) for the particular case $V = 200$ m/sec, $\eta_{th} = 0.45$, $\eta_h = 0.75$, $a = 58.72$, $L = 48,000,000$ (in M.K.S. units)

Assuming the efficiency of expansion ρ_d in the gas turbine equal to 0.90, formula (190) gives $v_0 = 920$ m/sec. The speed $\psi = 200/920 = 0.2175$ and the propulsive efficiency η_p is, by (194),

$$\eta_p = 1.171 \left(0.75 + \frac{0.0473}{\phi^2} \right) \rho_1$$

with, by (186)

$$\rho_1 = 2\phi^2 \left(\sqrt{\frac{0.9 + \phi^2}{0.0473 + \phi^2}} - 1 \right)$$

Thus by varying $\phi = U_e/v_0$ between 0 and 0.5, the values of ρ_1 , $\eta_h \rho_1$, $\rho_1 \frac{\psi^2}{\phi^2}$, and η_p were obtained and posted in table XXXII.

²³Formula (175) permits, however, an approximation not utilized above. The ratio $\frac{a+1}{a}$ was actually compared to unity. This simplification is entirely legitimate, as will be shown later on.

Table XXXII

Propeller Driven Exclusively by Reaction.

$$V = 200 \text{ m/sec} \quad \eta_{th} = 0.45 \quad \eta_h = 0.75 \quad a = 58.72 \quad v_0 = 920 \text{ m/sec}$$

$\varphi = U_e/v_0 \quad . .$	0	0.1	0.2	0.3	0.4	0.5	∞
$U_e \text{ m/sec} \quad . . .$	0	92	184	276	368	460	∞
$\rho_1 \quad$	0	0.0597	0.1825	0.3035	0.4035	0.483	0.853
$\eta_h \rho_1 \quad$	0	0.0478	0.1367	0.2274	0.3024	0.3622	0.64
$\frac{\psi^2}{\varphi^2} \rho_1 \quad$	0.318	0.282	0.2156	0.159	0.1193	0.0911	0
$\left(\eta_h + \frac{\psi^2}{\varphi^2}\right) \rho_1 \quad . .$	0.318	0.3298	0.3523	0.3864	0.4217	0.4533	0.64
$\eta_p \quad$	0.3725	0.3862	0.413	0.453	0.494	0.531	$\eta_p = \eta_h = 0.75$

The above values of η_p are in good agreement with those in table XXX and were calculated by the approximate formula (175) in which the ratio $(1 + a)/a = 59.72/58.72 = 1.017$ is compared to unity. The difference does not exceed 1 percent, hence justifies the approximation for simplifying (175).

The results of the preceding table are plotted in figure 47. The relatively slow increase of η_p with U_e is the result of the slowness with which the internal efficiency ρ_1 of the gas turbine increases with the speed of rotation $\varphi = U_e/v_0$ of this system.

62. Jet propeller realizing the best combination of driving and rotating system

As indicated at the end of article 54, the optimum mixed system, that is, giving the most advantageous combination of primary and rotating system, is now considered.

It is possible to determine (cf. article 50) the value h_2 of the coefficient of division h , which, for a given value of the parameter $Q = \eta_{th} L/av^2$, gives the maximum of the propulsive efficiency η_p a maximum which is then easy to calculate.

In order to study, by exact examples, the results that may be hoped for from this optimum mixed system, the question actually involves the determination of the best probable values of the thermal efficiency η_{th} of the cycle as well as the corresponding values of Q .

Having discussed at length the case of a mixed cycle (article 53) which involved a normal airplane engine, a revolving exhaust supplied by low or high pressure with respect to the normal exhaust pressure of such a system in (53), there remains the case of a cycle comprising, in theory, a combustion at constant pressure, inserted between the preliminary compression and a subsequent continuous expansion, both adiabatic, a cycle defined in (54).

Obviously all systems of the so-called balanced type, (articles 56 to 60) studied previously on the true jet propeller, can be employed here on the condition of recovering in the engine an additional portion of the expansion of the burnt gases in the form of effective mechanical energy that is transmitted by the said engine to the rotating system, the portion which is extracted at the actually produced expansion in the rotating system.

It is therefore immediately seen that, at preliminary compression and combustion produced in identical conditions, the second system gives a higher or lower thermal efficiency η_{th} than that of the first, depending upon whether the efficiency of expansion²⁴ of the supplementary expansion produced in the system is higher or lower than the value which it has when this expansion is produced in the revolving system.

In this last case, the truly optimistic value, $p_d = 0.9$ had been consistently admitted. It certainly is not possible to visualize a higher value for the same portion of expansion in the case where the latter is realized in the system itself instead of in the revolving unit.

This expansion efficiency could even be reduced to 0.85 or more when the engine is of the reciprocating type, and to 0.80 or more when of the turbine type.

²⁴It concerns here the efficiency of the real expansion as defined in (61) and, in particular, by the general formula (178), state (d) and (e) denoting the extreme state of real expansion under consideration, that is, in the present case, of the partial real expansion.

In these conditions (cf. table XIX) the conclusion is reached that, depending upon whether the coefficient h passes from zero (propeller driven by jet exclusively) to one (propeller driven by engine only), the values of η_{th} , indicated in this table, must be multiplied by a reduction factor ranging between 1 and $0.85/0.90 = 0.945$ (reciprocating engine), or between 1 and $0.8/0.9 = 0.89$ (turbojet), the values of the compression ratio λ_c and of the dilution α remaining practically the same.

For the previously cited reasons (increase in volume and decrease in the temperature of the gases in contact with the components in continuous rotative motion), the most favorable dilution α to be retained appears to be of the order of magnitude of 3, that is, for the Rey kerosene, a corresponding air volume of the order of 60 ($\alpha = 3$ corresponds in fact to $a = 58.72$).

In these conditions, it is seen that, if propulsive speeds of more than 200 m/sec are excluded and the best thermal efficiency η_{th} susceptible to give the best over-all efficiency $\eta_g = \eta_{th} \times \eta_p$ is retained, that is, an η_{th} of the order of 0.45 to 0.425 (if h passes from zero to unity), the minimum of the parameter Q to be visualized is of the order of

$$Q = \frac{\eta_{th} \times L}{a \times v^2} = \frac{(0.45 \text{ to } 0.425) \times 48,000,000}{58.72 \times 200 \times 200}$$

or of the order of 9.21 to 8.66 (h varying from zero to unity).

Formula (163) shows then that the corresponding maximum possible improvement of propulsive efficiency with respect to that of the propeller (which is the propulsive efficiency of the propeller driven by the engine exclusively, the exhaust being fixed, having but a negligible propulsive effect as in standard engine-propeller systems) does not exceed

$$\Delta = \left(\frac{0.3}{0.7}\right)^2 \frac{1}{2 \times 8.66} = 1.06 \text{ percent}$$

which is entirely negligible.

It is therefore superfluous to seek the best combination, which is practically very near to the standard engine-propeller system (fixed exhaust), and which without appreciable benefit, introduces the complications and difficulties inherent in a revolving exhaust.

Returning to the turbojet, there is some hope of being able to realize complete expansion (specific advantage of the internal combustion turbine), the great dilution ($\alpha = 3$) remaining, moreover, indispensable. But the thermal efficiencies obtainable drop then to values of the order of 0.24 to 0.25 maximum, which is below that of good airplane engines (0.27) and considerably below that of fuel injection engines which can reach 0.36.

63. Optimum orientation of exhaust jets

In the study of the general system represented in figure 34, it had been admitted that the exhaust of the rockets is exactly aligned along the absolute speed of the center of the discharge opening of these rockets, which describes a helicoidal trajectory.

It is expedient to investigate this orientation, in order to obtain the most advantageous orientation, that is, the maximum propulsive efficiency η_p .

Consider figure 48, in which the inclination γ_e of the exhaust jet of a rocket (equivalent to the cluster of rockets disposed around the axis of rotation) is different from the orientation β_e of the absolute speed of the said rocket.

To reconcile the free flow that flows around the envelope of the rocket with the jet that leaves the latter, it is necessary to give this envelope a somewhat special form, such as that indicated by figure 48, for example.

It should also be noted that the angles β_e and γ_e could be different from each other without entailing a considerable development of the envelope and an aerodynamic resistance of the latter that can no longer be justly neglected, as stated before and as applied to what follows.

To allow for the angle γ_e , simply revert to the calculations in articles 44 to 47 and replace β_e by γ_e every time the inclination of the corresponding speed of exhaust appears.

Thus formula (140) becomes

$$\eta_{th}^L = \frac{1}{m} C_h w_h + \frac{1+a}{2} (w_2^2 + U_e^2 - 2U_e w_e \sin \gamma_e) - \frac{aV^2}{2} \quad (195)$$

while formula (141) can be written as

$$T = T_h + m [(1+a)w_e \cos \gamma_e - aV] \quad (196)$$

Lastly, formula (144) modified, becomes

$$\frac{1}{m} C_h W_h = h \eta_{th} L + (1 + a)(w_e - W_e) \sin \gamma_e U_e \quad (197)$$

Therefore

$$x_e = \frac{w_e}{V} = \sqrt{\left[1 + 2(1 - h)Q\right] \frac{a}{1 + a} - \left[\tan^2 \beta_e - \frac{\sin \gamma_e \sin \beta_e}{\cos^2 \beta_e}\right]} \quad (198)$$

Finally, by the definition (9) of η_p , this efficiency can be expressed by the formula

$$\eta_p = h \eta_h + \frac{1}{Q} \left\{ (\cos \gamma_e + \eta_h \tan \beta_e \sin \gamma_e) \times \right. \\ \left. \sqrt{\frac{1 + a}{a} \left[1 + 2(1 - h)Q\right] + \left(\frac{1 + a}{a}\right)^2 \tan^2 \beta_e \left[2 \frac{\sin \gamma_e}{\sin \beta_e} - 1\right]} - \right. \\ \left. \left(1 + \eta_h \frac{1 + a}{a} \tan^2 \beta_e \frac{\sin \gamma_e}{\sin \beta_e}\right) \right\} \quad (199)$$

which corresponds to formula (152), when $\gamma_e = \beta_e$.

According to a legitimate conventional approximation, $(a + 1)/a$ is compared to unity; hence the preceding formula becomes

$$\eta_p = h \eta_h + \frac{1}{Q} \left\{ x_e \cos \gamma_e (1 + \eta_h \tan^2 \beta_e \tan \gamma_e) - \left(1 + \eta_h \tan^2 \beta_e \frac{\sin \gamma_e}{\sin \beta_e}\right) \right\} \quad (200)$$

with

$$x_e = \sqrt{1 + 2(1 - h)Q + \tan^2 \beta_e \left(\frac{2 \sin \gamma_e}{\sin \beta_e} - 1\right)} \quad (201)$$

formulas which permit the same approximations to be made as those used in the preceding numerical calculations and which can be obtained also by making $\xi = (a + 1)/a = 1$ in formulas (150) and (152).

It should be remembered that the formulas (200) and (201), in which the aerodynamic resistance of the rockets is not taken into account, may permit an insufficient approximation in the case where the angles β_e and γ_e differ considerably from each other.

64. Application to the specific case of the propeller driven exclusively by reaction

With formulas (200) and (201) and by making $h = 0$, the case of the propeller driven exclusively by reaction is obtained. Supposing that η_h , η_{th} , Q , and $\tan \beta_e$ are given and the problem is that of finding the optimum value of $\tan \beta_e$, that is, that which gives, in the conditions considered, the maximum η_p (and at the same time, η_g).

This value of $\tan \beta_e$ should cancel the derivative of η_p with respect to $\tan \gamma_e$, this quantity being taken as sole variable in (200).

The looked-for value is then found as the root of the equation

$$\tan \beta_e (1 - \tan^2 \gamma_e) \left(1 + \eta_h \tan \beta_e \tan \gamma_e \sqrt{\frac{1 + \tan^2 \beta_e}{1 + \tan^2 \gamma_e}} \right) + x_e \eta_h \tan \beta_e \left(x_e - \sqrt{1 + \tan^2 \beta_e} \right) - x_e^2 \tan \gamma_e = 0 \quad (202)$$

x_e being expressed by formula (201).

It is readily seen that this equation cannot, as a rule, be solved very easily, and requires a special study in each particular case. In consequence, we return to the particularly interesting case contemplated in article (60) (table XXX, fig. 44) and corresponding to

$$\eta_{th} = 0.45 \quad \eta_h = 0.75 \quad V = 200 \text{ m/sec} \quad Q = 9.21$$

Rather than solving (200) for different $\tan \beta_e = U_e/V$, it is more convenient to compute η_p for different values of $\tan \gamma_e$ and in each case, for different values of $\tan \beta_e$ by (200) and (201).

The results are represented in table XXXIII and figure 49.

Table XXXIII

Propeller Driven Exclusively by Reaction ($\eta_{th} = 0.45$; $\eta_h = 0.75$;

$V = 200$ m/sec; $Q = 9.21$) Rockets with Variable Orientation

	Values of η_p				
	$\tan \beta_e = 0$ $U_e = 0$ m/sec	0.5 100	1 200	1.5 300	2 400
$\tan \gamma_e = 0$	$\eta_p = 0.370$	0.367	0.358	0.342	0.318
0.5	.319	.382	.434	.465	.477
1	.229	.330	.416	.483	.527
1.5	.156	.276	.378	.462	.524
2	.105	.230	.340	.430	.507

Figure 49 shows the efficiency curve η_p of the propeller with normally oriented rockets, $\beta_e = \gamma_e$, studied in article (60).

The envelope of the curves ($\tan \gamma_e$) gives the maximum η_p for each value of $\tan \beta_e$, or U_e , and the corresponding orientation γ_e is the quota of the curve which touches the envelope at the point under consideration.

An examination of this figure shows that a certain improvement could be obtained if γ_e were given a value different from β_e , but this improvement does not exceed 2 percent in relative value and is therefore of secondary importance.

The difference between γ_e and β_e , in the optimum arrangement that affords this improvement, is readily apparent in figure 50, which represents the quoted curves ($\tan \beta_e$) with $\tan \gamma_e$ as abscissa, instead of the converse representation used in figure 49. It is seen that in the optimum arrangement ($\tan \beta_e - \tan \gamma_e$) increases from zero to about unity when $\tan \beta_e$ increases from 0 to 2. The corresponding difference ($\beta_e - \gamma_e$) therefore does not exceed 20 degrees, which should be near the permissible limit, in order that the employed formulas maintain a satisfactory degree of approximation. One advantage, although trifling, is afforded from this summary outline, namely, giving the inclination of the rockets on the axis of rotation a value smaller than the inclination of their absolute speed on the same axis. In a certain measure, this conclusion can be reached by the rule that obliges to give to the zero lift axis of a propeller blade section a geometrical pitch superior to that of the helicoidal trajectory of the said section.

In the corresponding position ($\gamma_e < \beta_e$) represented in figure 49, the reaction R_f due to the jet of gases leaving the rocket is itself inclined with respect to the speed V at an angle δ_e inferior to γ_e .

It can be shown, in effect, that the angle δ_e is given, in the case of the pure reaction propeller, by the simple formula

$$\tan \delta_e = \frac{x_e \sin \gamma_e - \tan \beta_e}{x_e \cos \gamma_e - 1} \quad (203)$$

which necessarily involves

$$\delta_e < \gamma_e \quad \text{if} \quad \gamma_e < \beta_e$$

$$\delta_e > \gamma_e \quad \text{if} \quad \gamma_e > \beta_e$$

65. Notes on the inherent advantage with a propeller driven exclusively by reaction

Such a propeller can be regarded as a combination of a true rocket (with liquid fuel), the jet of which instead of assuring direct propulsion by axial reaction (as in the case of the rockets studied in chapter 1 of Part B of the first part of this report) furnishes in part the propulsive thrust by means of a propeller which itself is driven by the peripheral (or tangential) component of the reaction supported by the exhaust nozzle, placed at tips of the blades and sensibly oriented along its own trajectory.

This combination permits, as already demonstrated, speeds of the order of 700 km/h at least, so as to obtain (by virtue especially of an improved thermal efficiency η_{th}) an important over-all efficiency η_g .

Moreover, the improvement of the thermal efficiency of this combination makes it possible to obtain a quasi-continuous and total expansion, effected in part in the balanced engine and in part in the revolving unit; it offers the advantage of suppressing every mechanical connection between the engine and the propeller, since, by definition, no transmission of mechanical energy is effected between these elements.

In consequence, a variation of the operating conditions of the propeller independent of the speed of engine itself can be provided. For example, a simple modification of the orientation of the exhaust jets (connected or not with a change of the propeller pitch) can be secured without modifying the speed of rotation of the adjoined engine.

This advantage appears to merit attention.

By the same argument, the principle of the propeller driven exclusively by reaction appears to be able to give rise in the form of a hydraulic transmission by jet (and consequently, in open circuit) between an energy generator and one or even several propulsive propellers, with certain interesting applications for the navy.

66. Addition of a thrust augments tube to the exhaust of the jet propeller

As envisaged for the true rockets with direct and axial reaction, the addition of a single or multiple thrust augments tube to the revolving exhaust nozzles can be conceived.

The general formulas necessary are easily extended to include this system, by reverting to the line of reasoning of articles (44) to (47) while bearing in mind that:

(1) At induction, the system traps the corresponding volumes of air a and a' , at entrance in the heat engine and, at entrance in the augments tube, respectively, the air in question may be regarded in one and the other case as being at absolute rest and under the conditions p_a, T_a of the still atmosphere.

(2) At evacuation, the system supplies a mixture of burnt gases and air trapped by the thrust augments tube, a mixture assumed homogeneous ($p_e = p_a, T_e$) and moving at a uniform relative speed w_e .

It is immediately apparent that the previously established formulas held true, provided that a is replaced by $(a + a')$.

If it is expedient to neglect the aerodynamic resistance of the rockets, fitted now with thrust augments tubes, and to adopt the approximation, even more legitimate here, namely, to assimilate the ratio $(a + a' + 1)/(a + a')$ to unity. It gives the following formulas, identical to (200) and (201), for the case where the orientation γ_e of the rocket can differ a little from the orientation β_e of the absolute speed of this rocket:

$$\eta_p = h\eta_h + \frac{1}{Q} \left\{ x_e \cos \gamma_e (1 + \eta_h \tan \beta_e \tan \gamma_e) - \left(1 + \eta_h \tan^2 \beta_e \frac{\sin \gamma_e}{\sin \beta_e} \right) \right\} \quad (203)$$

$$x_e = \frac{w_e}{V} = \sqrt{1 + 2(1 - h)Q + \tan^2 \beta_e \left(\frac{2 \sin \gamma_e}{\sin \beta_e} - 1 \right)} \quad (204)$$

with

$$Q = \frac{\eta_{th} L}{(a + a') V^2} \quad (205)$$

the thermal efficiency η_{th} applying, to be sure, to the complete thermal cycle, which includes the operation of the rockets and the thrust augments tubes.

When a jet propeller with normally oriented rockets ($\beta_e = \gamma_e$) is involved, it is sufficient to make $\gamma_e = \beta_e$ in the preceding equations and formulas which are omitted here since they are identical with those used in all the foregoing numerical calculations relative to jet propellers, except that $(a + a')$ is substituted for (a) .

To study the advantages accruing from the addition of thrust augments tubes to the exhaust of a normal jet propeller, it is necessary to compare the over-all efficiency η_g of the jet propeller with thrust augments tubes with that of the jet propeller without augments tubes to which it corresponds, the only difference between the two arrangements consisting in the arrangement of the exhaust nozzles.

With a double accent denoting the quantities of the reference jet propeller, without augments tube, the problem consists in ascertaining if and to what extent the ratio

$$X = \frac{\eta_g}{\eta_g''} = \frac{\eta_{th}}{\eta_{th}''} \times \frac{\eta_p}{\eta_p''} \quad (206)$$

can exceed unity; the relative difference is a measure of the advantage or benefit accruing from the addition of augments tubes.

In this comparison $L, a, V, \eta_{th} \tan \beta = U_e/V$ are obviously taken identical in both cases.

In these conditions η_p'' and η_p are solely dependent on the corresponding values of the parameter Q , or

$$Q'' = \frac{\eta_{th}'' L}{aV^2} \quad (207)$$

$$Q = \frac{\eta_{th}^L}{aV^2} = Q'' \times \frac{\eta_{th}}{\eta_{th}''} \times \frac{a}{a + a'} \quad (208)$$

The problem is one of knowing how the ratio

$$Y = \frac{\eta_{th}}{\eta_{th}''}$$

varies as function of the ratio $\mu = \frac{a'}{a}$

As previously pointed out, Y depends, in the particular conditions, chiefly on μ and the corresponding pressure $\lambda_1 = p_1/p$ in the mixer of the thrust augments tube (when a single, adiabatic augments tube is involved), or in other words, Y decreases when Q increases, irrespective of λ_1 .

On the other hand, it is easily checked that η_p decreases when Q increases.

Therefore, when μ increases (starting from zero, case of exhaust without effect of thrust augments tube), $Y/(1 + \mu)$ decreases, that is, Q decreases and, consequently, η_p increases. The variation of η_p then results in variations, in the opposite sense, of η_{th} or Y and η_p .

There is absolutely nothing to prevent thinking that η_g could, in these conditions, increase and therefore the addition of augments tubes afford a greater or lesser benefit.

But every test of the numerical calculation will necessitate assumptions for which the experimental basis, which is indispensable, is utterly lacking at the present time.

For more details on this subject, the reader is therefore referred to the chapters of the second part of this report devoted to rockets with thrust augments tubes, by stressing once more the importance of a

systematic experimental study of the operation of gas augments tubes and the influence of μ and of λ_1 on the thermal efficiency of the cycle of active bodies of fluids in thrust augments tubes of this kind.

CHAPTER II

Summary and Conclusions of Part II

67. Review of results obtained in chapter I

The general scheme of the propulsive system represented by figure 34 involves the combination of a heat engine, in which a more or less important part of the complete thermodynamic cycle of the active bodies takes place, and a propeller which at the same time constitutes a revolving exhaust, in which the aforementioned cycle is achieved.

The propeller is driven, in general, by the torque transmitted by the engine on its shaft and by the latter operating as a gas turbine.

The over-all thrust is the sum of the propeller thrust and the axial component of the reactions due to the exhaust jets.

The envisaged arrangement includes the usual engine-propeller combinations, the rockets with axial and direct reaction and, lastly, the true jet propeller.

The principal results may be summed up as follows:

The formulas in articles 44 to 47 constitute the fundamental formulas. They lend themselves to legitimate simplifications so that the propulsive efficiency η_p can be expressed by the formulas (153) and (155).

When the rockets are inclined with respect to their trajectory, these formulas can be replaced by (200) and (201), which may cease to be applicable if the inclination γ_e of the rockets differs too much from the β_e of their trajectory.

Assuming the propeller efficiency η_h and the inclination $\tan \beta_e = U_e/V$ of helicoidal trajectory of the revolving rockets on the axis of rotation as given, the efficiency η_p depends only on the coefficient h (defined in article 44, formula (138)) and on the parameter $Q = \eta_{th} L/av^2$, and it is, in particular, a decreasing function of Q (article 50).

Within the possible scope of application and at speed not exceeding 700 to 800 km/h, η_p passes through a maximum (η_h , $\tan \beta_e$, and Q being constants) at a certain value of h slightly below unity and this maximum exceeds the propeller efficiency η_h very little.

In other words, in these particular conditions, the combination of engine and the most advantageous rotating system is practically identical to the standard engine-propeller system where h is equal to unity, that is, the effective energy of the thermodynamic cycle (defined in article 44) entirely realized in form of mechanical energy in the engine and transmitted by the latter to the propeller (cf. article 51).

The greatest attention should be given to the obtainable value of the thermal efficiency η_{th} (and to the corresponding consumption of air a , since this term affects both the efficiency η_p and the over-all efficiency $\eta_g = \eta_{th} \times \eta_p$, which definitively constitutes the true criterion of valuation (cf. article 52).

When proceeding from an ordinary reciprocal engine and fitting it with a revolving exhaust without modifying the pressure of combustion or the volumetric ratio of expansion in the engine cylinders, the over-all efficiency with respect to the engine-propeller system taken for starting point is reduced (article 53).

When it is expedient to look for new and substantially different types of heat engines (reciprocal or rotary) to form the engine, the thermal efficiency η_{th} with respect to the actual aircraft engines, can be improved by trying to secure a complete and continuous expansion of the burnt gases in spite of their passage from the engine to the revolving system (article 54).

The extreme case of a self-acting electric propeller, assuring the centrifugal compression of the air at the inside of the suitably hollow blades and comprising combustion chambers and expansion nozzles at the blade tips, is of no practical interest by reason of the low η_{th} , due to insufficient compression (article 56).

The arrangement of the true jet propeller (or driven exclusively by reaction) which permits a balanced engine ($h = 0$) is of real interest theoretically, for it promises considerable improvement of η_{th} (article 59, table XXIX) and at the same time affords certain advantages due to the elimination of every mechanical connection between engine and propeller (article 61).

However, this arrangement is inferior to that of the optimum system (corresponding to a certain value of h near unity), which functions with the same value of the ratio η_{th}/a .

For the true jet propeller, two main types of engine can be envisaged: either a balanced reciprocal engine (with pistons) with rather high compression ratio, or a balanced turbojet with lower compression ratio, these two types of engines with quasi-continuous and complete expansion to operate with a rather substantial dilution α . The first actually preponderates and is used to envisage applications with over-all efficiency perhaps equivalent or superior to that of present-day engine-propeller systems, provided that the speed is in excess of 700 to 800 km/h (article 60).

The second is much inferior to the first at the present state of engineering of turbomachines (compressors and turbines).

As concerns the relatively slow decrease of η_p as function of $\tan \beta_e$, that is, of the peripheral speed, the properties of the true jet propeller are directly connected to those of the single rotor gas turbine (cf. article 61).

The addition of thrust augmentation to the exhaust of the jet propeller (article 66) should not be condemned a priori. For a profitable study of this question, the various problems concerning gas augmentation, about which there is a lack of data, must be subjected to systematic experimentation.

68. Classification of the various interesting types of propulsive systems - ways of progress

The present report deals with propulsive systems which produce their propulsive effect either exclusively by direct reaction, or by direct (axial or helicoidal) and indirect reaction simultaneously, the latter being obtained by making a propeller act in the outside fluid.

Three main systems are of interest:

(a) The classical engine-propeller combination, which sensibly corresponds to the case ($h = 1$, $\tan \beta_e = 0$) of the system termed jet propeller.

(b) The system or propeller driven exclusively by reaction and balanced engine of the reciprocal or rotary type, a system corresponding to case $h = 0$ of the reaction propellers.

(c) The system called pure rocket or rocket with thrust augmentation, with direct axial reaction.

Summarily, it may be stated that the second and third appear to be able to produce an efficiency equivalent or superior to that actually obtained by the first, provided that the propeller speed exceeds 700 to 800 km/h in the second case, and 1200 to 1500 km/h in the third.

Considering that the propeller efficiency can decrease rather substantially at high and very high speeds of propulsion, the last two systems appear destined to supplant the propeller in these speed ranges, the first of the systems in question assuring so to say, the transition between propeller and rocket.

Although these speed ranges are of no use for the present, the research concerning those systems of propulsion present a certain interest for the future.

From this point of view, we believe that the research to be undertaken should be concentrated on:

The improvement of the efficiency of the compression and expansion turbines, which involves that of the specific horsepower and the efficiency of heat engines with piston, and which strive toward their ultimate form, that is, toward the internal combustion turbine.

The operation of gas augmenters, a problem still insufficiently explored, offers a multitude of interesting applications.

APPENDIX I

Steady Flow of Viscous Gases

1. General equation of steady flow.

A gas (a compressible fluid whose inside state is defined by the two variables: density ρ and temperature T) can be expressed by the general equations of steady²⁵ motion

$$\left. \begin{aligned} \frac{1}{\rho} \frac{\partial p}{\partial x} &= X - j_x + \frac{\zeta + \eta}{\rho} \frac{\partial \theta}{\partial x} + \frac{\eta}{\rho} \Delta u \\ \dots \dots \dots & \\ \dots \dots \dots & \end{aligned} \right\} \quad (1)$$

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0 \quad (2)$$

where p represents the "reversible pressure" corresponding to the state (ρ, T) of the fluid by the equation of state or of compressibility, the vector (j_x, j_y, j_z) of the acceleration of the fluid at the point in question, the parameter θ the cubic expansion $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$, and ζ and η the coefficients of viscosity of Navier and Poisson, which depend on (ρ, T)

In addition, there is the equation of state

$$f(p, \rho, T) = 0 \quad (3)$$

These five equations are insufficient for determining the six unknown functions: u, v, w, p, ρ, T , to which must be added a supplementary relation which can be supplied only by thermodynamics.

2. Supplementary relation.

It is necessary to introduce the properties of the fluid with respect to the propagation of the heat (by conduction and radiation) in its mass.

²⁵The steady motion, that is, with constant speed at each point, which excludes the agitation which, if it occurs, can be allowed for by the introduction of a fictitious viscosity of turbulence.

The radiation is disregarded (although it is not always negligible for real gases when they are at elevated temperatures).

Ordinarily, the introduction of the conductivity is eliminated by assuming it zero or infinite.

The internal conductivity of real gases being, in general, slight, its effect may be disregarded in first approximation in the rapid transformations of these fluids.

Considering only the rapid transformations of a gas endowed with viscosity and flowing at high speed in steady state, the conductivity is assumed to be negligible.

Therefore, on considering an element of mass dm of such a fluid, the heat $d'Q$ which it receives from the outside in time interval dt , and which is reduced to that transmitted by the surrounding fluid by conductivity, is zero. Hence, according to the Carnot-Clausius principle

$$d'Q = (TdS + d'C_{vi}) dm = 0 \quad (4)$$

S signifying the entropy per unit mass and $d'C_{vi}$ the energy of the internal viscosity in time interval dt per unit mass.

This energy of the internal passive resistance is

$$d'C_{vi} = -\frac{dt}{\rho} \left\{ \xi \theta^2 + 2\eta \left[\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial z} \right)^2 \right] + \right. \\ \left. \eta \left[\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \right)^2 + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right)^2 \right] \right\} \quad (5)$$

Suppose, on the other hand, that the gas obeys the laws of perfect gases in the states of equilibrium

$$\left. \begin{aligned} p &= R\rho T \\ dU &= cdt \\ R &= Cte = C - c \\ c &= \text{function of } T \end{aligned} \right\} \quad (6)$$

The principles of thermodynamics give for these laws

$$TdS = d\left(U + \frac{p}{\rho}\right) - \frac{dp}{\rho} \quad (7)$$

Thus the desired supplementary relation (4) takes the form (with due allowance for (5) and (6))

$$\frac{d}{dt}\left(U + \frac{p}{\rho}\right) - \frac{1}{\rho} \frac{dp}{dt} = - \frac{1}{\rho} \left[\xi \theta^2 + 2\eta A^2 + \eta B^2 \right] \quad (8)$$

with

$$A^2 = \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2$$

$$B^2 = \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)^2 + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2$$

3. Equation of flow along a filament.

In the case under consideration the principle of the conservation of energy does not supply, in general, the first integral of the motion along a flow filament.

By suppressing the viscosity, the first integral of the motion along a steady filament is (in the absence of forces at distance $X = Y = Z = 0$)

$$U + \frac{p}{\rho} + \frac{V^2}{2} = \text{Cte} \quad (9)$$

hence a relation analogous to that by Bernoulli (first integral of the kinetic energy) in the case of incompressible fluid ($\rho = \text{Cte}$) and devoid of viscosity, in steady flow

$$\frac{p}{\rho} = \frac{V^2}{2} = \text{Cte} \quad (10)$$

However, an equation similar to (9) along a steady filament of viscous and nonconducting gas can be obtained.

From the first three equations (1) (where $X = Y = Z = 0$ by assumption), follows, after multiplying each equation by udt , vdt , wdt and adding

$$\frac{dp}{\rho} = -d \frac{V^2}{2} + \frac{\xi + \eta}{\rho} d\theta + \frac{\eta}{\rho} \Delta \bar{V} \bar{ds} \quad (11)$$

where \bar{V} is the speed (u, v, w), $\Delta \bar{V}$ the vector ($\Delta u, \Delta v, \Delta w$), and \bar{ds} the displacement (udt, vdt, wdt) along the filament.

Reconciling (11) with (8), gives the desired relation

$$d\left(U + \frac{p}{\rho} + \frac{V^2}{2}\right) = \frac{1}{V} \left[\frac{\xi}{\rho} \theta^2 + \frac{\eta}{\rho} (2A^2 + B^2) \right] ds + \frac{\xi + \eta}{\rho} d\theta + \frac{\eta}{\rho} \Delta \bar{V} \bar{ds} \quad (12)$$

or

$$d\left(U + \frac{p}{\rho} + \frac{V^2}{2}\right) = -d'C_{vi} + \frac{\xi + \eta}{\rho} d\theta + \frac{\eta}{\rho} \Delta \bar{V} \bar{ds}$$

The derivative of the function $\left(U + \frac{p}{\rho} + \frac{V^2}{2}\right)$ with respect to the displacement s along the filament is not necessarily negative (the term $-d'C_{vi}$ is essentially positive), contrary to what is at times believed when establishing a wrong comparison between the second term of (12) and the loss of head of viscous fluids.

Equation (12) expresses the application of the principle of the conservation of energy (or first law of thermodynamics) to the flow along the steady flow filament. The direct application of this principle introduces the energy of the normal forces (which differ from the reversible pressure p) on the straight sections upstream and downstream from the portion of the filament in question, as well as the energy of the tangential actions of the ambient fluid on the lateral surface of the filament portion. The foregoing reasoning avoids the difficulty of evaluation of these reports by the use of formula (5) of the energy $d'C_{vi}$ of the internal viscosity, a formula assumed as known and which derives from the fundamental notions which thermodynamics form on the viscosity.

4. Note.- Comparison with incompressible fluid.

In the case of a liquid ($\rho = \text{Cte}$) the relation

$$d\left(\frac{p}{\rho} + \frac{V^2}{2}\right) = \frac{\eta}{\rho} \Delta \bar{V} \bar{ds} \quad (13)$$

along a steady flow filament and in absence of remote external forces is easily obtained from the equations of motion without having recourse to thermodynamics.

This relation cannot be considered as a particular case of (12), where $\xi = 0$ and $\theta = \text{Cte}$ for the viscous and incompressible fluid, as the latter is a poor conductor and experiences an adiabatic transformation as explicitly assumed when establishing (12).

But the properties of a viscous fluid involve the relation

$$T dS = dU$$

and, therefore, by calling $d'Q$ the quantity of heat received by conductivity in time interval dt by an element immersed in the mass, the quantity referred to unit mass is

$$d'Q = T dS + d'C_{vi} = dU + d'C_{vi} \quad (14)$$

In the case of viscous liquid, this relation and (13) constitute two general equations for the flow along a steady filament. If the liquid is a very poor conductor, $d'Q$ is negligible, and it is seen that relation (12) applied to the specific case of the liquid whose flow is steady and essentially adiabatic

$$d\left(U + \frac{p}{\rho} + \frac{v^2}{2}\right) = -d'C_{vi} + \frac{\eta}{\rho} \Delta \bar{V} \bar{ds}$$

actually decomposes in two simultaneously verified relations

$$d\left(\frac{p}{\rho} + \frac{v^2}{2}\right) = \frac{\eta}{\rho} \Delta \bar{V} \bar{ds}$$

$$dU = -d'C_{vi}$$

The first of these two relations, that is, relation (13), is moreover verified whatever the thermodynamic transformation of the fluid, since it is established direct, starting from the four equations of motion and the equation of the state of the liquid, $\rho = \text{Cte}$. However, it is dependent upon the nature of this transformation, because the coefficient of viscosity η , which depends only on temperature T , varies actually with the latter, in general.

In the case of the liquid the transformation is generally regarded as isothermal ($T = \text{Cte}$); either the liquid is a very good conductor and in relation with a single source of heat or the viscosity is very low and the liquid without calorific communication with the outside medium.

In such cases, η is practically constant and the equations of motion (as well as the general relation (13)) do not introduce the temperature other than by the constant value to be given to η . This is the study of motion without recourse to thermodynamics.

5. Steady and sensibly cylindrical flow of viscous gas.

By steady and sensibly cylindrical motion is meant the steady flow in which the speed is practically reduced to its component parallel to a fixed axis, such as axis x , for instance, and in which, moreover, the speed and the state of the fluid are uniform on every parallel, that is, on every circle orthogonal to the axis of the flow and centered on this axis.

Every plane perpendicular to the axis x of the flow constitutes then an orthogonal section (straight section of flow) and the reversible pressure p_y is constant.

Always assuming the gas to be devoid of internal conductivity and disregarding the radiation, the equation of motion (x being taken as variable abscissa and r as distance from the axis) is reduced to

$$\frac{1}{\rho(x,r)} \frac{\partial p}{\partial x} = -u(x,r) \frac{\partial u}{\partial x} + \frac{\xi + 2\eta}{\rho} \frac{\partial^2 u}{\partial x^2} + \frac{\eta}{\rho} \left(\frac{1}{r} \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial r^2} \right) \quad (15)$$

$$u \frac{\partial \rho}{\partial x} + \rho \frac{\partial u}{\partial x} = 0 \quad (16)$$

with the unknown functions $u(x,r)$, $p(x)$, and $\rho(x,r)$ and the coefficients of viscosity ξ and η dependent on ρ and on T .

The equation of state being assumed to be that of perfect gases

$$p = R\rho T \quad (17)$$

the foregoing equation must be augmented by the supplementary relation (8) and which can be written as

$$\frac{1}{\gamma - 1} \frac{u}{\rho} \frac{\partial p}{\partial x} + \frac{\gamma}{\gamma - 1} \frac{p}{\rho} \frac{\partial u}{\partial x} = - \frac{\xi + 2\eta}{\rho} \left(\frac{\partial u}{\partial x} \right)^2 - \frac{\eta}{\rho} \left(\frac{\partial u}{\partial x} \right)^2 \quad (18)$$

The equations (15) to (18) are the complete equations of motion which are used for computing the unknown u , p , ρ , and T when the initial and the extreme conditions are known.

APPENDIX II

On the Theory of Viscous Fluids in Nozzles

1. The problem.

Consider the steady flow of a viscous fluid in a nozzle fitted on the wall of an infinite tank.

The theoretical problem consists in determining in complete manner either the flow corresponding to a given nozzle or the nozzle in correspondence with a given flow.

The theory of nozzles is as follows: The apparently steady real flow is replaced by a viscous fluid, moving or otherwise, and whose internal tensions and mean speed are variable in every straight section of the flow, the flow by sections, that is, with uniform speed and temperature in every section perpendicular to the axis of flow, of a non-viscous flow obeying the same equation of state, sliding against the wall and submitting, in part of it, to a reaction of retarding friction. This retarding action represents the effect of internal viscosity of the real fluid as well as the tangential forces in contact with latter and with the wall. The nature of the thermodynamic transformation undergone by this fictitious fluid is expressed by a simple hypothesis.

2. Approximate theory of nozzles.

The equations of flow are easily put in the following form; the subscripts 1 and 2 refer to any two straight sections (assumed plane):

$$\frac{V_2^2 - V_1^2}{2} = - \int_1^2 \sigma dp - \int_1^2 X dx \quad (\text{equation of motion}) \quad (1)$$

$$\left(\frac{\omega V}{\sigma} \right)_1 = \left(\frac{\omega V}{\sigma} \right)_2 = m \quad (\text{equation of continuity}) \quad (2)$$

Quantity X designates the absolute value of the retarding action referred to unit mass, x the displacement along the stream, ω the area of the straight section, and m the volume of the nozzle by mass.

Quantity X is usually put in form of

$$X = \eta \frac{X}{\omega} V^2$$

X designating the perimeter of contact (wetted perimeter) of the fluid with the wall in the straight section of area ω . In the case of circular nozzles

$$X = \pi D \quad \text{and} \quad \omega = \frac{\pi D^2}{4}$$

we get

$$X = \eta \frac{4V^2}{D} \quad (3)$$

With the force X (per unit of mass) expressed in this form, the coefficient η is nondimensional. By virtue of the homogeneity of the equations this function η is thus a function, with numerical coefficients, of the invariants, the number of which is determined by the Vaschy theorem after all independent quantities affecting the value of X are listed.

(a) Case of incompressible fluid.- This fluid being characterized by $\sigma = \text{Cte}$, equations (1) and (2) are adequate for solving the problem of plotting the nozzles once it is known to express the coefficient η of formula (3).

Ordinarily η is put equal to Cte , the choice of numerical values being obtained from test data.

(b) Case of compressible fluid.- Equations (1) and (2) are no longer sufficient because, x being taken for variable and m being given, they contain four functions p, σ, ω, V of x , only one of which is given theoretically [$\omega(x)$ when the nozzle is given, $p(x)$ when the flow is given].

A supplementary relation must be obtained from thermodynamics. Moreover, except when neither this relation nor the expression of η introduce the temperature, the equation of state of the fluid must be introduced

$$p = f(\sigma, T) \quad (4)$$

The supplementary relation corresponding to the fictitious flow is set forth in the following manner.

The quantity of heat $^{26}\delta m d'Q$ received by the fluid element of mass δm during its path dx is the sum of:

(a) The heat volume $\delta m d'Q_i$ transmitted to the particular element by the surrounding fluid by radiation and conduction

(b) The heat volume $\delta m d'Q_t$ transmitted to this element by the nozzle through radiation and conduction, or heat exchange with the wall by convection

(c) Lastly, the part $(1 - \alpha)(0 \leq \alpha \leq 1)$, taken by this fluid element of the heat volume equivalent to the work of friction of the fluid against the nozzle, $\delta m d'Q_f$.

Dividing by δm and referred to unit mass of fluid, we get

$$d'Q = d'Q_i + d'Q_t + (1 - \alpha)d'Q_f$$

On the other hand, by the principle of Carnot-Clausius

$$d'Q = TdS = dU + pd\sigma$$

so the supplementary relation takes the form

$$TdS = dU + pd\sigma = d'Q_i + d'Q_t + (1 - \alpha)d'Q_f \quad (5)$$

To establish a convenient connection between the conventional flow of the fictitious fluid and the real flow, certain appropriate assumptions can be made about the exchanges of heat $d'Q_i$, $d'Q_t$, $(1 - \alpha)d'Q_f$ which concern the fictitious fluid.

However, when assuming that the flow is accomplished by sections and of a uniform temperature in every straight section of the stream, the hypothesis is ostensibly simplified, as it is useful to specify that the heat volume ($\delta m d'Q$) received by the section of the fluid of mass δm and thickness $\delta x = \frac{\delta m}{\omega \sigma}$ is either zero or else uniformly distributed in the mass.

²⁶The accent given to the letter d is a reminder that the quantity $d'Q$ is not, in general, a total differential.

On the other hand, it is natural or reasonable to admit for most problems of flow in nozzles that the nozzle itself exchanges only a negligible amount of heat with the outer atmosphere, that is to say, that the total heat volume which it receives from the fluid $(-d'Q_t + \alpha d'Q_f)$ is zero.

The total heat volume $d'Q$ received per unit mass of fluid and assumedly uniformly distributed in the transverse thickness of the stream a is then

$$d'Q = d'Q_i + d'Q_t + (1 - \alpha)d'Q_f = d'Q_i + d'Q_f$$

For this quantity of heat, which penetrates in the section, of mass δm and thickness δx , through its outside surface (straight sections upstream, downstream, and laterally in contact with the nozzle), to be uniformly distributed in the transverse thickness of the section, it is necessary to assume

(a) That the fluid is a perfect conductor, in which case nothing permits to predict the value of $d'Q_f$

(b) Or that the fluid is devoid of conductivity and hence, $d'Q_f = \text{zero}$ and, since $d'Q_f$, is not zero, (it is immediately seen that $d'Q_f$ is equal to Xdx or $\frac{4\eta}{D} V^2 dx$), it must be conceded that the quantity of heat $d'Q$ is uniformly transmitted in its entire transverse thickness of the section other than by conductivity, as by radiation, for instance.

Notwithstanding the difficulties raised by this line of reasoning, it is admitted here that, by analogy with the real fluid, the conductivity of the fictitious fluid must be regarded as negligible, so that

$$d'Q_i = 0$$

$$d'Q = d'Q_f = \frac{4\eta}{D} V^2 dx$$

In these conditions, the desired supplementary relation is

$$TdS = dU + pd\sigma = \frac{4\eta}{D} V^2 dx \quad (6)$$

The final equations of the fictitious flow are as follows:

$$\left. \begin{aligned} (a) \quad VdV &= -\sigma dp - \frac{4\eta}{D} V^2 dx \text{ (equation of motion)} \\ (b) \quad \omega V &= m\sigma \text{ (equation of continuity)} \\ (c) \quad p &= f(\sigma, T) \text{ (equation of state)} \\ (d) \quad TdS &= dU + pd\sigma = \frac{4\eta}{D} V^2 dx \text{ (supplementary relation)} \end{aligned} \right\} \quad (I)$$

The equation (Ia) expresses what may be called the equation of the active forces for the fictitious fluid, derived direct from the equations of motion.

Replacing it by the relation that supplies the principle of the conservation of energy, we get

$$-d(p\sigma) - \frac{4\eta}{D} V^2 dx + d'Q = dU + VdV$$

This equation, together with (Id), is equivalent to equation (Ia), but expressed in different form.

3. Gas nozzles:

With the equations (I) established for the flow of the fictitious gas and assuming that this gas obeys the laws of perfect gases (R being a constant and C a simple function of T):

$$p\sigma = RT = (C - c)dT$$

$$TdS = dU + pd\sigma = CdT - \sigma dp$$

The equations of flow become

$$\left. \begin{aligned} VdV &= -\sigma dp - \frac{4\eta}{D} V^2 dx \\ \omega V &= m\sigma \\ p\sigma &= RT \\ CdT - \sigma dp &= \frac{4\eta}{D} V^2 dx \end{aligned} \right\} \quad (II)$$

In these conditions the problem of the gas nozzle would involve either (1) the function $\omega(x)$ (that is, the form of the nozzle) being given for the determination of the functions V, p, σ, T of x (that is, the flow) or (2) one of the functions V, p, σ, T of x being given (that is, the law of flow) for the determination of $\omega(x)$ and the other unknown functions (that is, the form of the nozzle and the complimentary characteristics of the flow).

The preceding four equations (I) enable a complete solution of this problem provided it is known how to express η as function of x .

In incompressible fluid, the approximation $\eta = Cte$ is often resorted to. For a compressible gas or fluid, the case under consideration here, such an assumption would be too roughly approximate to give acceptable results and η should be considered as variant with V, p, σ , and T as well as with $\frac{dp}{dx}$, because the effect of the internal viscosity of a real fluid is different, as proved by experiment, depending on whether the flow is accompanied by expansion (expansion nozzle) or by compression (compression or diffuser nozzle).

Therefore, the whole problem of gas nozzles, reduced to a fictitious problem according to the foregoing conception, consists in determining the form of the function η which characterizes the retarding action acting on fictitious fluid.

In spite of considerable researches, this problem cannot be said to be solved, and is in need of systematic experimental study.

The equations (II), of course, are applicable only in the absence of shock waves in the fluid (across a wave of this kind, the supplementary relation set up above ceases to be applicable and should be replaced by another such as the law of Hugoniot or the law of dynamic adiabatic), that is to say, as long as the speed of flow is, at the most, equal to the velocity of sound in the fluid at the same point.

4. Velocity in the throat of a nozzle.

One important result relative to the velocity in the throat of the nozzle, when the flow satisfies the equations (II) is the following:

By differentiation of the equation of continuity and assuming $d\omega$ negligible with respect to dV and $d\sigma$, the velocity in the thrust is

$$\sigma_c dV = V_c d\sigma \quad (7)$$

Subscript c refers to the throat of the nozzle.

On the other hand, the first of the equations (II) can be written as

$$V_c dV = -\sigma_c dp \left[1 + \frac{4\eta}{D} V_c^2 \left(\frac{dx}{dp} \right)_c \right] \quad (8)$$

Dividing (8) by (7) and reducing, leaves

$$V_c = \sqrt{-\sigma_c^2 \left(\frac{dp}{d\sigma} \right)_c \left[1 + \frac{4\eta}{D} V_c^2 \left(\frac{dx}{dp} \right)_c \right]} \quad (9)$$

The transformation of the fluid, in the throat as along the nozzle, is an endothermic reaction which can be expressed by

$$-\left(\sigma^2 \frac{dp}{d\sigma} \right)_c = \frac{C}{c} (p\sigma)_c - \frac{R}{c} \sigma_c d'Q_f = \gamma (p\sigma)_c - (\gamma - 1) \left(\sigma \frac{d'Q_f}{d\sigma} \right)_c$$

The velocity of sound a_c in the fluid at the throat of the nozzle is given by the classical formula

$$a_c^2 = \gamma (p\sigma)_c$$

so that equation (9) can be written as

$$V_c = \sqrt{\left[a_c^2 - (\gamma - 1) \left(\frac{\sigma d'Q_f}{d\sigma} \right)_c \right] \left[1 + \left(\frac{d'Q_f}{dp} \right)_c \right]} \quad (10)$$

When $d'Q_f$ is essentially positive, the preceding formula shows that, if expansion occurs in the throat of a nozzle, (dp negative, $d\sigma$ positive), we get

$$V_c < a_c$$

or, in other words, that the velocity of a fluid at the throat is lower than the velocity of sound in the fluid at the same point.

It is remarked that, provided that $d\omega$ is negligible in the throat with respect to dp and $d\sigma$, this result is rigorous because the equations (II) are valid for the throat since a shock wave cannot exist in this point where V_c is lower than a_c .

Furthermore, if the expansion of the fluid is continued upstream from the throat, no shock wave can exist upstream from the throat (the convergent part of the nozzle), since in this area the velocity of sound decreases continuously when the speed of flow increases continuously without being able to reach the first.

In the case of a compression nozzle or diffuser, equation (10) shows that, $d\sigma$ being negative and dp positive

$$V_c > a_c$$

that is, the speed of the fluid in the throat of the diffuser exceeds the velocity of sound at that point. Moreover, a shock wave can form upstream from the throat.

5. Representation in Mollier diagram.

Suppose that the state of the fluid at each instant and per unit mass is represented in the Mollier diagram where S is taken as the abscissa and the total heat of the fluid $\Lambda = U + p\sigma$ as the ordinate.

For a gas that obeys the laws of perfect gases, this diagram differs from the entropy diagram only by a change of the scale of the ordinates since, for a gas of this kind

$$d\Lambda = d(U + p\sigma) = C(T)dT$$

On the Mollier diagram, like on the entropy chart, the lines of constant pressure are parallel curves derived by a simple translation along the axis (S) of the abscissa. The slope of the tangent to a line of constant pressure is equal to the absolute temperature in each point.

Let (1) be the curve representing (fig. 51) the transformation of the gas along flow starting from point M_1 representing the state of the gas in a reference section 1.

The system nozzle-gas undergoes, with respect to the outside atmosphere at uniform temperature Θ , a monothermal transformation during which, according to the Carnot-Clausius theorem, the noncompensated work, that is, the loss of energy, has for elementary value d'_{II} with respect to unit mass

$$d'_{II} = \Theta dS \tag{11}$$

since the exchange of heat with the outside is zero and the nozzle does not rub against the surrounding atmosphere.

On the other hand, the supplementary relation (5) is written

$$d'Q_f = \frac{4\eta}{D} V^2 dx = T dS \quad (12)$$

hence the loss of energy

$$d'II = \frac{\Theta}{T} d'Q_f = \frac{\Theta}{T} \frac{4\eta}{D} V^2 dx \quad (13)$$

This loss, essentially positive, represents the decrease in effective energy of the fluid in consequence of irreversibilities which the flow introduced in the nozzle. This effective energy \underline{r} (with exception of a constant of no significance since variations in \underline{r} are only considered) is represented by the function

$$\underline{r} = U + p\sigma - \Theta S + \frac{V^2}{2} = \Lambda - \Theta S + \frac{V^2}{2}$$

Between two sections 1 and 2 of the flow the laws of thermodynamics give

$$\frac{V_2^2 - V_1^2}{2} = \Lambda_1 - \Lambda_2 + \Theta(S_2 - S_1) - \int_1^2 d'II \quad (14)$$

The Mollier diagram affords a convenient representation of the preceding quantities when, as assumed on figure 51, the curve (1) representing the transformation of the fluid can be plotted in it.

Draw through the point M_1 the straight line $\Delta\Theta$ of slope Θ (parallel to the tangent of the line of equal pressure p_1 at the point of the temperature Θ , taken here arbitrarily as origin of the coordinates S and Λ).

It is readily seen, according to (11) and (14), that for a point M_2 on the line of transformation (1) and along the scale of Λ :

The vertical distance $\overline{\mu_2\mu_2}$ represents the total energy loss (positive) $II_{1,2}$ from 1 to 2;

The vertical distance $\overline{M_2\mu_2}$ represents the change in kinetic energy $\Delta W_{1,2} = \frac{1}{2}(V_2^2 - V_1^2)$ from 1 to 2.

The tangent to the line of transformation (λ) has the angular coefficient

$$\left(\frac{d\lambda}{dS}\right)_\lambda = \left(\frac{CdT}{C \frac{dT}{T} - R \frac{dp}{p}}\right)_\lambda = T_\lambda \frac{1}{1 - \left(\frac{\sigma}{C} \frac{dp}{dT}\right)_\lambda}$$

Let us examine the case of expansion nozzles and that of compression nozzles, considering only the parts of the flow without shock wave, so as to preserve the validity of the flow equations (II).

However, it should be noted that in both cases the entropy S always increases by crossing along the flow, since $TdS = d'Q_f$ (essentially positive); the line of transformation (λ) is therefore always passed in the sense of increasing entropy.

(a) Expansion nozzles.- By definition dp is, in this case, always negative.

Starting from a point M_1 without speed, $\Delta W = \frac{V_2^2}{2}$ can only be positive at the point M_2 of the pressure p_2 below p_1 and, consequently, the point M_2 on the isobar p_2 can only be situated between the points A_2 and B_2 , and the line (λ) must fall within the curvilinear triangle $A_2M_1B_2$ without having any vertical tangent.

If the speed always tends toward increasing, the curve (λ) decreases from left to right and vice versa (as seen from the relation $VdV = -CdT$).

The shape of the curve (λ) is largely dependent upon the form of the coefficient η in the expression of $d'Q_f$, that is, the importance of the effects of viscosity and friction in each section of the real flow, effects which are summarily represented by the retarding effect acting on the fictitious fluid.

Therefore, an increase in speed requires an expansion, but the reciprocal is not true, that is, the acceleration in an expansion nozzle is not necessarily always positive; in particular, the fluid may, without ceasing to expand, undergo a deceleration.

(b) Compression nozzles.- By definition, dp is, in this case, invariably positive.

Hence, since $dS = C \frac{dT}{T} - R \frac{dp}{p}$ is essentially positive, dT is necessarily positive and dV negative.

The line of transformation (1) always ascends from left to right and presents neither vertical nor horizontal tangent, as exemplified by the curve M_1M_3 in figure 51.

So the compression nozzle always produces a deceleration of the flow, but the reciprocal is not true, as seen from the rise for the expansion nozzle. It being understood that the compression nozzle functions only from an initial state M_1 of the stream at speed other than zero and only up to the pressure for which this initial kinetic energy is canceled, that is, up to a final temperature T_f such that

$$\int_{T_1}^{T_f} C_d T = \frac{V_1^2}{2} \quad (15)$$

Starting from a state without speed, the flow is thus simply readied by the expansion.

The foregoing results arise principally from the assumed existence of the fictitious retarding action η or the wall friction.

When this friction disappears, the transformation of the fluid is adiabatic and reversible, hence defined by

$$T dS = C_d T - RT \frac{dp}{p} = 0$$

and represented by the vertical in point M_1 . Expansion and compression will therefore be rigorously synonyms of acceleration and deceleration of the flow.

6. Nonadiabatic nozzles.

In the foregoing, the exchange of heat across the sections of the flow ($d'Q_i = 0$) as well as between the nozzles and the outside atmosphere Θ had been assumed zero.

These assumptions are generally admitted and appear adequate for the majority of problems on gas nozzles with sufficiently high speed.

Nevertheless, there are problems for which these assumptions must be modified, such as when the nozzle is systematically subjected to cooling or heating from the outside.

In cases of that kind it is expedient, while conserving, if necessary, the assumption $d'Q_i = 0$, to put

$$d'Q_t - \alpha d'Q_f = d'Q_e \neq 0$$

$d'Q_e$ represents the quantity of heat supplied to the nozzle from the outside.

The equations of flow then read:

$$\left. \begin{aligned} VdV &= -\sigma dp - \frac{4\pi}{D} V^2 dx \\ \omega V &= m\sigma \\ p\sigma &= RT \\ CdT - \sigma dp &= \frac{4\eta}{D} V^2 dx + d'Q_e \end{aligned} \right\} \quad (III)$$

the energy loss becomes

$$d'II = \Theta dS - d'Q_e = \frac{\Theta}{T} d'Q_f - \left(1 - \frac{\Theta}{T}\right) d'Q_e$$

$\Theta < T$, if $d'Q_e$ is negative and vice versa.

The expansion nozzles are again classified as heated nozzles and cooled nozzles. The term $d'Q_e$ is readily allowed for when representing on the Mollier diagram the quantities ΔW and II which compensate the variation $\Delta(\Lambda - \Theta S)$ in the energy balance of the flow.

7. A frequently utilized approximation.

The complete approximate theory of nozzles requires the knowledge of the fictitious retarding action, that is, the coefficient η which characterizes it as function of the characteristics of the fictitious stream (p, σ, T, V, ω) and of the state of the wall in that particular section.

For compressible fluids or gases, the theory is still incomplete on this point.

At times this difficulty can be avoided by assuming a priori a certain law for the transformation experienced by the fluid, a law called the polytropic law and of the form

$$p\sigma^n = C^{te} \quad \text{with } n > 1 \quad (16)$$

This relation forms the supplementary relation. Together with the equations of motion, continuity, and state, it supplies the system

$$\left. \begin{aligned} VdV &= -CdT \text{ (equation of motion)} \\ \omega V &= m\sigma \text{ (equation of continuity)} \\ p\sigma &= RT \text{ (equation of state)} \\ p\sigma^n &= Cte \text{ (supplementary relation)} \end{aligned} \right\} \quad (IV)$$

of four equations where the abscissa x of the particular section does no longer intervene.

With p taken as variable, the four unknown functions σ , T , ω and V of p , can then be determined from the foregoing system.

Of course, this does not mean the exact shape of the nozzle, (that is, the area ω of each section as function of abscissa x); the correct length must be defined by experiment. Let us examine the value which involves the phenomena of viscosity and friction of the real flow for the exponent n of the polytropic law of the fictitious fluid when the nozzle is assumed adiabatic with respect to the outside medium.

If the fictitious flow was reversible, the supplementary relation becomes

$$p\sigma^\gamma = c^{te}$$

γ is the ratio of the specific heat C/c at constant pressure and constant volume.

In fact, the fictitious flow is irreversible and it has been shown elsewhere that dS is then invariably positive, which can be expressed by the condition

$$dS = \left[\frac{n-1}{n} C - R \right] \frac{dp}{p} > 0$$

Hence the following conclusions:

(a) Expansion nozzles.- The flow is continuously accelerated, since, by virtue of the equations (IV) and the exponent n being essentially greater than unity, dp and dT are consistently negative.

Moreover, $n < \gamma$ of necessity and therefore

$$1 < n < \gamma$$

Hence, for a given ratio $\frac{p_2}{p_1}$ of expansion, the increase in kinetic energy and the drop in temperature are less than if the expansion were reversible.

Furthermore

$$\left. \begin{aligned} v_2^2 &= v_1^2 + \frac{2\gamma}{\gamma - 1} p_1 v_1 \left[1 - \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} \right] \\ T_2 &= T_1 \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} \end{aligned} \right\} \quad (17)$$

(b) Compression nozzles.- For these nozzles the flow is consistently decelerated and

$$n > \gamma$$

which causes, for a given ratio of compression $\frac{p_2}{p_1}$, the decrease in kinetic energy and the increase in temperature to be much greater than if the expansion were reversible. The formulas (17) are still applicable, but the exponent n has then a value greater than γ .

APPENDIX III

The Theory of Thrust Augmenters, and Particularly
of Gas Augmenters

1. Definition of thrust augmenters.

A thrust augmenter is a jet apparatus consisting of fixed elements in form of nozzles and in which a primary fluid entrains another fluid by transferring part of its effective energy to it.

This transmission of energy may have for principal object to increase either the pressure of the entrained fluid or its kinetic energy. The Giffard steam injector is an example of a thrust augmenter of the first category, whereas the rocket with thrust augmentation belongs to the second category.

The single thrust augmenter tube corresponds to the diagrammatic sketch of figure 52. The driving fluid m , launched by nozzle A, entrains the fluid m' , conveyed by nozzle B, in the part of the nozzle C called "mixer", then in the diffuser D (generally divergent).

The nozzles can have any desired sectional shape. Generally of circular section to favor symmetrical flow and facilitate the machining of the walls, they occasionally present a rectangular section giving jets in waves.

By multiplying the inflow nozzles of the fluid, as indicated in figure 53, a multiple thrust augmenter is obtained. The purpose of this arrangement is to favor the progressive entrainment of the passive fluid by the active fluid by increasing the areas of mutual contact of the fluid jets actuated at different speeds. For the same reason, the jet of the driving fluid is, theoretically, centric and not outside, but it may also be given an annular section in certain applications so as to render its external and internal surfaces active simultaneously.

2. Approximate theory of augmenters.

As to the flow in the inlet and discharge nozzles, the ordinary concepts of the nozzle theory developed in the preceding appendix can be assumed. But it should also be noted that the latter are not applicable to the discharge nozzle or diffuser if the flow in latter is comparable to a homogeneous fluid mixture.

To avoid any difficulty in this respect, it is advisable to restrict, by definition, the part of the thrust augmenter called "mixer", on the upstream side to the straight section²⁷, to the point of incipient contact of the driving fluid with the entrained fluid on the downstream side to the straight section starting at the point where the temperature and the speed can be compared to uniform quantities in every straight section of flow.

As to the functioning of the thus defined mixer, two principal conceptions can be visualized:

The driving fluid and the entrained fluid are regarded as forming two distinct flows, that is, do not mix, and friction, viscosity, and exchange of heat prevail at the surface of contact or else the two fluids are considered as intermingling, with friction, viscosity (internal by contact and diffusion), and exchange of heat introduced in such a way as to produce a homogeneous mixture acting as a single fluid downstream from the mixer.

The first concept, that is, that of augmenters with individual flows can be developed in a theory, but it is not applicable to devices with rapid and turbulent jets where the mixture of the jets is manifest, experimentally. Moreover, it raises certain difficulties resulting from the arbitrary assumption that the speed and temperature in a certain section which limits, by definition, the mixer downstream are simultaneously equal in the two flows. Lastly, even if this objection did not exist, the flow in the diffuser would not be formed, a priori, in the same manner for the contiguous flows and will give rise to contact effects similar to those produced in the mixer.

The second concept, that is, that of thrust augmenters with flows effectively mixed in the mixer itself, responds much better to reality. It has been utilized, particularly by Rateau (Theory of Augmenters by the author, published in the Revue de Mecanique, 1900, Paris; see also: Cours de Machines, taught at the Ecole Nationale Supérieure des Mines by E. Jouguet) in his classical study, and which is adopted in the present study, restricted to the single thrust augmenter.

3. Assumptions on the functioning of the mixer:

According to the concept to be adopted, the motions of the real fluids cannot be reduced to those of fictitious fluids devoid of

²⁷The single nozzle is involved here. For multiple nozzles, the sections of incipient contact of flows serve as entrance sections in the mixer unit.

viscosity because internal and contact viscosity play an essential part in making the velocities uniform in the mixer.

The effected simplification is limited to the real functioning, for the formulation of the approximate theory in mind.

It is assumed that:

(a) The viscosity exerts no appreciable effect in the inlet and discharge sections of the mixer.

(b) Speed and temperature are uniform in the two sections of entry and in the common discharge section.

(c) The fluid mixture leaving the mixer is homogeneous.

(d) The regime of functioning is steady.

(e) The mixer is adiabatic, that is, transfers no heat to the outside.

(f) The reversible pressure that exists at every point of the fluids contained in the mixer is constant.

The last assumption merits some comment.

On the one hand, it is evidently approximative and entirely justified by the simplifications which it effects on the theory.

It affords, at least, a summary and simple study of the functioning.

On the other hand, this pressure in the mixer is identical with the actual pressure existing according to (a) at the inlet and outlet of the mixer. At the lateral surface of the mixer, that is, against the wall, it represents the normal force exerted by the fluid against the said wall. But, as concerns the tangential force along this wall, it cannot be supposed that it is everywhere zero, since friction and contact viscosity of the fluid which give rise to tangential forces are not, a priori, negligible and it is not certain that the speed of the fluid along the wall is everywhere negligible, hence, that the boundary layer adheres.

It should be noted that the previously enumerated assumptions are definitely not incompatible.

That the viscosity may have an indifferent effect in the extreme sections is a common assumption for regular flows to which those

entering and leaving the mixer are compared. That the final mixture is homogeneous, that is, the elements of the fluid mixtures are distributed at each point with the same temperature for two contiguous elements and under uniform pressure with respect to identical masses and volumes, is a physically plausible assumption.

That the velocities and temperatures are uniform in the two inlet sections and in the common outlet section is a simplified representation of the real problem which simply consists in defining these velocities and temperatures by their mean values for the corresponding volumes and assuming that the mean difference in velocity with respect to the above mentioned average is small or, in other words, that the velocity is sensibly uniform in the extreme sections so that the average of the thus defined velocities can be used to express the average kinetic energy.

Lastly, that the (reversible) pressure of the fluids contained in the mixer is uniform, that is, constant in the entire mass, is the sole assumption which particularizes the functioning at the inside of the mixer. Without conceding that it could give rise to certain reservations, it is admitted here for the purpose of simplifying the problem, as already adopted by various authors, and particularly by Rateau.

4. Equations of functioning of the mixer.

The corresponding quantities of the entrained fluid are accented to distinguish them from those of the actuating fluid, subscripts 1 and 2 refer to the sections at entry and exit.

The volumes by mass of the actuating and the entrained fluid are denoted by m and m' , the corresponding partial inlet and outlet sections by ω and ω' (the extreme total sections are $\Omega_1 = \omega_1 + \omega_1'$ and $\Omega_2 = \omega_2 + \omega_2'$).

Considering gas augmenters only, it is assumed that the gases obey the laws of perfect gases in the reversible transportations.

The sole object of the equations to be formulated is to link the conditions (sections, volumes, speeds, and states of both fluids) at the mixer outlet to the corresponding conditions at the mixer inlet.

As in all problems of the mechanics of compressible fluids, hydrodynamics supplies the equations of motion (reduced to only one since it concerns, in the extreme sections, flows at speeds parallel to a specified direction, that of the axis of flow), and the equations of continuity for the two fluids supplemented by the equation of state of

these fluids and a supplementary relation which involves thermodynamics and which expresses the nature of the transformation undergone by the fluids in the mixer.

The equations of continuity are expressed by the relations

$$\left. \begin{aligned} m\sigma_1 &= \omega_1 w_1 & m'\sigma_1' &= \omega_1' w_1' \\ m\sigma_2 &= \omega_2 w_2 & m'\sigma_2' &= \omega_2' w_2 \end{aligned} \right\} \quad (1)$$

since, by assumptions, $w_2' = w_2$.

The equations of state read

$$\left. \begin{aligned} p_1 \sigma_1 &= RT_1 & p_1 \sigma_1' &= R'T_1' \\ p_1 \sigma_2 &= RT_2 & p_1 \sigma_2' &= R'T_2 \end{aligned} \right\} \quad (2)$$

where R and R' denote constants, since, by assumption

$$p_2' = p_2 = p_1' = p_1 \quad \text{and} \quad T_2' = T_2$$

To obtain the over-all relation corresponding to the equations of motion, the momentum theorem which serves to establish the equations of hydrodynamics can be applied to the fluids contained in the mixer. Projected on the axis of the mixer and with F_t as the absolute value of the resultant forces (opposite to the direction of flow), the tangential forces exerted by the wall of the mixer on the adjacent fluid are

$$(m + m')w_2 - mw_1 - m'w_1' = -F_t$$

or

$$(m + m')w_2 = (1 - j)(mw_1 + m'w_1') \quad (3)$$

j denoting a positive number which expresses the effect of these tangential stresses as will be explained later.

The last relation is obtained by indicating that the functioning of the mixer is adiabatic with respect to the outside.

The application of the Carnot-Clausius principle in unit time to the mixer and to the mixing fluids gives

$$\Sigma \delta m \int_1^2 T dS + \Sigma \delta m' \int_1^2 T' dS' = -\Delta C_i$$

the sum Σ being extended to the corresponding volumes m and m' , and ΔC_i denoting the total and essentially negative energy of the effects of viscosity and friction, in unit time internal as well as by contact between fluids or with the wall, introduced in the mixer.

For the gases under consideration

$$T dS = C dT - \sigma dp$$

and, by virtue of the assumption of constancy of pressure in the entire mixer ($p \equiv p_1$), $T dS = C dT$ or with Λ denoting the total heat, that is, the function defined for perfect gases by $d\Lambda = d(U + p\sigma) = C dT$

$$T dS = d\Lambda$$

Finally, the supplementary relation reads

$$m(\Lambda_2 - \Lambda_1) + m'(\Lambda_2' - \Lambda_1') = -\Delta C_i \quad (4)$$

Λ_2' refers to the same temperature (T_2) as Λ_2 .

The equation of motion can be obtained in a different form than (3) by applying the principle of the conservation of energy to the mixer and to the fluids contained in it. With the adiabatic capacity in respect to the outside taken into consideration

$$m(\Lambda_1 - \Lambda_2) + m'(\Lambda_1' - \Lambda_2') = (m + m') \frac{w_2^2}{2} - m \frac{w_1^2}{2} - m' \frac{w_1'^2}{2} \quad (5)$$

which, with (4) allowed for, gives

$$m \frac{w_1^2}{2} + m' \frac{w_1'^2}{2} - (m + m') \frac{w_2^2}{2} = -\Delta C_i \quad (6)$$

The term $-\Delta C_i$ (ΔC_i = total energy of internal effects of friction and of viscosity) being essentially positive like each of the terms of the first member of (6), it is apparent that this term is, at best, equal to the initial total energy of the fluid masses discharged through the mixer.

Hence

$$-\Delta C_1 = k \left[m \frac{w_1^2}{2} + m' \frac{w_1'^2}{2} \right] \quad (7)$$

the factor k (which depends on the operational conditions and especially on m , m' , w_1 , and w_1') being positive and equal to unity at the most.

The convention (7) is used to write the equation of motion in the form

$$(m + m')w_2^2 = (1 - k) \left[mw_1^2 + m'w_1'^2 \right] \quad (8)$$

This form should be equivalent to equation (3) obtained by utilizing the momentum theorem and this is utilized to link the factor j of equation (3) with the factor k defined by (7).

Therefore

$$1 - j = \sqrt{(1 - k) \left[1 + \frac{mm'(w_1 - w_1')^2}{(mw_1 + m'w_1')^2} \right]} \quad (9)$$

This relation shows that j is related to k by a rather complex expression and indicates the effect of the Carnot-Clausius principle on the value of j .

According to this principle, k ranges between zero and unity and cancels out only in the specific case where the actuating and the entrained fluids are identical and enter the mixer at the same speed and in the same state. In this event, $w_1' = w_1$ and j is zero like k .

In the limiting case where $k = 1$, j is also equal to unity.

Thus j and k introduced in the equation of motion in form of (3) or (8) range between zero and unity and are simultaneously equal to one another within these limits. Moreover, they are connected through the equality (9).

This remark offers an occasion to emphasize that, for the assumptions of this theory to be really compatible as affirmed previously, it is essential to admit the existence of a resultant of tangential stresses other than zero in contact with the wall and the fluid.

In the case of incompressible fluids, the equations of state are reduced to

$$\sigma = \text{constant} = \sigma_1$$

$$\sigma' = \text{constant} = \sigma_1'$$

while the equation of motion (3) is still applicable.

Quantity k can still be defined by (7), since ΔC_i always represents the work of the internal actions of friction and viscosity defined by the usual method of rational hydrodynamics.

Equation (9) is also still applicable. It expresses the equation of motion (3) in a different form and is derived direct from the kinetic energy equation.

Therefore j and k are still connected by equation (3).

In this case, it should be noted, equations (3), (8), and (9) are independent of any assumption regarding the exchange of heat; the transformation may be isothermal as well as adiabatic. An assumption of this kind is necessary only for determining the temperature changes of the fluids produced in the mixer, which may exert a certain influence on the value of the coefficient of viscosity and so, on the energy ΔC_i to which the factor k , which figures in (8) and (9), corresponds.

It will be recalled that, in his theory of thrust augmenters in incompressible fluids, Rateau confined himself to studying the form (3) of the equation of motion by giving a priori, to $(1 - j)$ a certain value below unity and considered (m, m', w_1, w_1') as nearly independent of the conditions of functioning of the mixer, at least within a certain range of these variables.

5. Energy losses in the mixer.

The mixer and the fluids contained in it being considered as a system in monothermal transformation (in contact with the atmosphere at temperature Θ), the energy loss II_m in the mixer in unit time is

$$II_m = \Theta \left[m(S_2 - S_1) + m'(S_2' - S_1') \right]$$

according to the Carnot-Clausius principle, and with the adiabatic process of the system with respect to the outside being allowed for; it can be written as

$$II_m = m \int_{T_1}^{T_2} d\Lambda \left(\frac{\Theta}{T} - 1 \right) + m' \int_{T_1}^{T_2} d\Lambda' \left(\frac{\Theta}{T} - 1 \right) + \\ m(\Lambda_2 - \Lambda_1) + m'(\Lambda_2' - \Lambda_1')$$

or by (5) as

$$II_m = m \int_{T_1}^{T_2} d\Lambda \left(\frac{\Theta}{T} - 1 \right) + m' \int_{T_1}^{T_2} d\Lambda' \left(\frac{\Theta}{T} - 1 \right) + \\ m \frac{w_1^2 - w_2^2}{2} + m' \frac{w_1'^2 - w_2'^2}{2} \quad (10)$$

The significance of the different terms in this expression is readily apparent. In the mixer the actuating fluid transfers heat irreversibly and at constant pressure to the entrained fluid. It is cooled between T_1 and T_2 , while the latter is heated between T_1' and T_2 and this transfer and the heat absorption induce, with respect to the source Θ , a loss which can be avoided by introducing Carnot cycles between the fluids and the source Θ , which afford the realization of effective energy, the algebraic value of which is represented by the first two terms of (10) and the sum of which is always positive.

Furthermore, the entrained fluid receives, according to (5) and (6) and the essentially negative sign of ΔC_i , more heat than it transfers to the driving fluid, the difference representing the decrease in total kinetic energy, which constitutes the second part, likewise always positive, of the energy loss represented by the two last terms of (10).

6. Equations of functioning of the complete thrust augmentor.

It is expedient to add to equations (1), (2), (7), and (8) (or in place of (8), (3) and (9)) established for the mixer, the equations of functioning of the inlet nozzles and of the diffuser.

The equations of the usual nozzles theory are taken in their over-all form corresponding to the whole of each nozzle.

Subscripts i and f refer to the extreme sections of the complete thrust augmenter; the quantities of the entrained fluids are distinguished by accents.

The inlet nozzles operate between p_i (or p_i') and p_1 , the diffuser between p_2 and p_f .

For simplification, it is assumed that the expansions and compressions of the fluid in the adiabatic nozzles are achieved by a polytropic law of the form

$$p\sigma^n = \text{cte}$$

the coefficient n ranging between 1 and $\gamma = C/c$ for expansion, higher than γ for compression.

Theoretically, n varies in a certain way with p depending upon nozzle and fluid, but to simplify matters it is assumed that for a nozzle with respect to total expansion or compression, n can be given a constant value when the ratio of the extreme pressures is modified a little.

In these conditions, the equations of functioning for each nozzle, between the extreme sections with subscripts q and r , assume the form

$$\left. \begin{aligned} V_r^2 - V_q^2 &= 2(\Lambda_q - \Lambda_r) \text{ (motion)} \\ m &= \left(\frac{\omega V}{\sigma}\right)_q = \left(\frac{\omega V}{\sigma}\right)_r \text{ (continuity)} \\ p_q \sigma_q^n &= p_r \sigma_r^n \text{ (supplementary relation)} \end{aligned} \right\} \quad (11)$$

and these relations can be utilized separately for the two inlet nozzles and for the diffuser.

The problem of the thrust augmenter is put as follows:

Given:

the volume m and m'

the initial state of the fluids: p_i, T_i (and σ_i) p_i', T_i' (and σ_i')

the initial speeds: w_i, w_i'

Find, when p_f is known, the final state T_f (and σ_f , σ_f') of the expelled mixture, its speed w_f as well as the characteristic sections of the mixer.

It is easily seen that the problem, as put, contains only one arbitrary quantity, namely, the pressure p_1 in the mixer, if suitable values can be given to n (exponent of the polytropic laws) and j or k (effects of viscosity and friction in the mixer).

In the following the equations of the gas augmenters are repeated with p and T expressed as variables of the state of the fluid, the specific volume σ being related to these variables by the equation of state $p\sigma = RT$ of perfect gases.

The equations defining the states and velocities are

$$\begin{array}{l}
 \left. \begin{array}{l} \text{Inlet} \\ \text{nozzles} \\ (I)a \end{array} \right\} \begin{cases} w_1^2 - w_i^2 = 2CT_i \left[1 - \left(\frac{p_1}{p_i} \right)^{\frac{n-1}{n}} \right] \\ w_1'^2 - w_i'^2 = 2C'T_i' \left[1 - \left(\frac{p_1}{p_i'} \right)^{\frac{n'-1}{n'}} \right] \\ \frac{T_1}{T_i} = \left(\frac{p_1}{p_i} \right)^{\frac{n-1}{n}} \\ \frac{T_1'}{T_i'} = \left(\frac{p_1}{p_i'} \right)^{\frac{n'-1}{n'}} \end{cases} \\
 \\
 \left. \begin{array}{l} \text{Mixer} \\ (I)m \end{array} \right\} \begin{cases} (m + m')w_2^2 = (1 - k) \left[mw_1^2 + m'w_1'^2 \right] \\ (m + m')\Lambda_2 - m\Lambda_1 - m'\Lambda_1' = k \left[mw_1^2 + m'w_1'^2 \right] \end{cases}
 \end{array}$$

$$\text{Diffuser (I)d} \left\{ \begin{array}{l} w_f^2 - w_2^2 = 2C_d T_2 \left[1 - \left(\frac{p_1}{p_f} \right)^{\frac{n_d}{n_d-1}} \right] \\ \frac{T_f}{T_2} = \left(\frac{p_f}{p_2} \right)^{\frac{n_d}{n_d-1}} \end{array} \right.$$

In these equations n_d is the polytropic exponent with respect to the transformation experienced by the homogeneous mixture circulating in the diffuser for which

$$C_d = \frac{mC + m'C'}{m + m'} \quad \gamma_d = \frac{m\gamma + m'\gamma'}{m + m'}$$

and

$$1 < n_d < \gamma_d \quad \text{at expansion}$$

$$\gamma_d < n_d \quad \text{at compression in the diffuser}$$

The equations defining the specific volumes in the extreme sections of the different parts of the thrust augments are

$$(II)a \quad \left\{ \begin{array}{l} p_1 \sigma_1 = RT_1 \\ p_1 \sigma_1' = R'T_1' \end{array} \right.$$

$$(II)m \quad \left\{ \begin{array}{l} p_1 \sigma_2 = RT_2 \\ p_1 \sigma_2' = R'T_2 \end{array} \right.$$

$$(II)d \quad \left\{ \begin{array}{l} p_f \sigma_f = R_d T_f \end{array} \right.$$

Lastly, the equations defining the partial sections ω or the total sections Ω are

$$(III)a \left\{ \begin{array}{l} \omega_i = m \frac{w_i}{\sigma_i} \quad \omega_l = m \frac{w_l}{\sigma_l} \\ \omega_i' = m' \frac{w_i'}{\sigma_i'} \quad \omega_l' = m' \frac{w_l'}{\sigma_l'} \\ \Omega_l = \omega_l + \omega_l' \end{array} \right.$$

$$(III)d \left\{ \begin{array}{l} \omega_2 = m \frac{w_2}{\sigma_2} \quad \omega_f = m \frac{w_f}{\sigma_f} \\ \omega_2' = m' \frac{w_2'}{\sigma_2'} \quad \omega_f' = m' \frac{w_f'}{\sigma_f'} \\ \Omega_2 = \omega_2 + \omega_2' \\ \Omega_f = \omega_f + \omega_f' \end{array} \right.$$

It is readily seen from these 24 equations containing 23 different quantities ($R, R', R_d, n, n', n_d, \gamma, \gamma', \gamma_d$, and k being assumed known) that all the unknowns can be determined when the nine quantities $m, m', p_i, p_i', T_i, T_i', w_i, w_i'$, and p_f are given, and the pressure p_l in the mixer is regarded as an arbitrary quantity as function of which the unknowns are expressed.

7. Energy efficiency of the thrust augments.

The energy efficiency of the augments, strictly speaking, is defined by the ratio of effective energy of the mixture delivered at the outlet to the effective energy of the introduced fluids. The difference between these two terms represents the energy loss ($II_a + II_m + II_d$) produced in the three parts of the thrust augments in unit time.

Considering only compressible fluids, their useful energy must be defined, in a certain state and per unit mass, by the difference of the values of the function

$$U - \Theta S + p\sigma + \frac{w^2}{2} = \Lambda - \Theta S + \frac{w^2}{2} \quad (12)$$

in the particular state and in a reference state for which, by definition, the effective energy is regarded as zero.

For an augments operating in the atmosphere, a fluid must be considered as being devoid of useful energy when it is at temperature Θ and atmospheric pressure p_a without speed ($w = 0$).

Therefore

$$\Lambda_{(p-\Theta)} - \Theta S_{(p_a, \Theta)} = 0$$

a relation which links the arbitrary constants in the general expression of the total heat Λ and of the entropy S .

The effective energy Γ is then

$$\left. \begin{aligned} \Gamma &= \int_{\Theta}^T d\Lambda - \Theta \left[\int_{\Theta}^T \frac{d\Lambda}{T} - \int_{p_a}^p R \frac{dp}{p} \right] + \frac{w^2}{2} \\ \Gamma &= \int_{\Theta}^T \left(1 - \frac{\Theta}{T} \right) d\Lambda + \Theta R \log \frac{p}{p_a} + \frac{w^2}{2} \end{aligned} \right\} \quad (13)$$

The energy efficiency of the complete thrust augments is

$$\rho_{en} = \frac{m\Gamma_f + m'\Gamma_f'}{m\Gamma_1 + m'\Gamma_1'} = 1 - \frac{II_a + II_m + II_d}{m\Gamma_1 + m'\Gamma_1'} \quad (14)$$

and the energy losses corresponding to the three sections (all three adiabatic) are

$$\begin{aligned} II_a &= \Theta [m(S_1 - S_1) + m'(S_1' - S_1')] \\ II_m &= \Theta [m(S_2 - S_1) + m'(S_2' - S_1')] \\ II_d &= \Theta [m(S_f - S_2) + m'(S_f' - S_2')] \end{aligned} \quad (15)$$

The values of the temperature, pressure, and entropy can be obtained by solving the equations (I) and (II) of the preceding article 6. This, in turn, is used to determine the efficiency and the energy losses in the thrust augments by the formulas (13), (14), and (15) in each particular case.

8. Special study of jet and gas thrust augmenters.

Only the states, speeds, efficiency, and losses of the system are considered.

The jet and gas thrust augments is a nozzle in which a volume m of actuating gas, starting without speed at a state (p_c, T_c) corresponding, in general, to the end of a combustion, is utilized to induce the entrainment of a certain volume of air m' , taken from the atmosphere in the conditions p_a, Θ and with a certain speed V , and to evacuate the mixture $(m + m')$ at pressure p_a and temperature T_E into the atmosphere with a discharge velocity w_E .

This particular category of jet apparatus in compressible fluids is tied to the general theory by

$$\begin{aligned} w_1 &= 0 & w_1' &= V & w_F &= w_E \\ p_1 &= p_c & p_1' &= p_a & p_F &= p_a \\ T_1 &= T_c & T_1' &= \Theta & T_F &= T_E \end{aligned}$$

The general equations are simplified by making the ratios nondimensional.

All the pressures are referred to p_a , all temperatures to T_c , and all speeds to V , and the following table of notations is adopted.

Notation:

$$\frac{m'}{m} = \mu; \quad \frac{p_c}{p_a} = \lambda_c; \quad \frac{p_1}{p_a} = \lambda_1; \quad \frac{T_c}{\Theta} = \theta_c; \quad \frac{T_1}{\Theta} = \theta_1; \quad \frac{T_1'}{\Theta} = \theta_1'; \quad \frac{T_2}{\Theta} = \theta_2;$$

$$\frac{T_E}{\Theta} = \theta_E; \quad \frac{w_1}{V} = \varphi_1; \quad \frac{w_1'}{V} = \varphi_1'; \quad \frac{w_2}{V} = \varphi_2; \quad \frac{w_E}{V} = \varphi_E; \quad \frac{R'}{R} = r; \quad \frac{C'}{C} = h$$

$$\frac{C_d}{C} = h_d; \quad \frac{n-1}{n} = s; \quad \frac{n_1'-1}{n_1'} = s'; \quad \frac{n_d-1}{n_d} = s_d$$

For simplification C and C' (and hence γ and γ') are regarded as constant quantities for the actuating and the entrained gases in question.

In these conditions, the equations of functioning (cf. equations (I) and (II), article 6) of the nozzles, of the mixer, and of the diffuser are, after simple reductions

$$\varphi_1^2 = 2\theta_c A \left[1 - \left(\frac{\lambda_1}{\lambda_c} \right)^s \right] \quad (16)$$

$$\varphi_1'^2 = 1 + 2hA \left[1 - \lambda_1^{s'} \right] \quad (17)$$

$$\varphi_2^2 = \frac{1-k}{1+\mu} \left[\varphi_1^2 + \mu \varphi_1'^2 \right] \quad (18)$$

$$\varphi_E^2 = \varphi_2^2 + 2h_d \theta_2 A \left[1 - \lambda_1^{-sd} \right] \quad (19)$$

$$\theta_1 \lambda_c^s = \theta_c \lambda_1^s \quad (20)$$

$$\theta_1' = \lambda_1^{s'} \quad (21)$$

$$\theta_2(1 + \mu h) - (\theta_1 + \mu h \theta_1') = \frac{k}{2A} (\varphi_1^2 + \mu \varphi_1'^2) \quad (22)$$

$$\theta_E \lambda_1^{sd} = \theta_2 \quad (23)$$

To simplify, take

$$A = \frac{C\Theta}{V^2}$$

It should be understood that, in all the formulas presented in this study, the quantities of heat and energy are assumed to be expressed with the same unit of energy. The ratio A is, therefore, a pure number.

Equations (16) to (23) are used to resolve the problem of the jet thrust augments such as visualized, that is, when

$$\frac{m'}{m} = \mu; \quad \Theta; \quad T_c = \theta_c \Theta; \quad p_a; \quad p_c = \lambda_c p_a; \quad p_1 = \lambda_1 p_a; \quad V$$

are given, and the coefficients or exponents $A, \gamma, h, h_d, s, s', s_d$, the eight equations above are used to determine the eight unknown quantities $\varphi_1, \varphi_1', \theta_1, \theta_1', \varphi_2, \theta_2, \varphi_E$, and θ_E which characterize the speeds and the states of the fluids at the principal points of the system.

The factor k is to be regarded as an assumedly known function of the nature of the fluids and the conditions $(\varphi_1, \varphi_1', \theta_1, \theta_1')$ at mixer entrance.

9. Energy efficiency of the jet and gas thrust augments.

The definition given in article 7 is applied.

The energy efficiency ρ_{en} of the thrust augments is

$$\rho_{en} = (1 + \mu h) \frac{\theta_E - (1 + \log \theta_E) + \frac{\varphi_E^2}{2A}}{\theta_c - (1 + \log \theta_c) + \frac{\gamma - 1}{\gamma} c \log \lambda + \frac{\mu h}{2A}} \quad (24)$$

This relation permits, when θ_E and φ_E are computed by the method indicated in the preceding article, the calculation of the efficiency of the thrust augments and the evaluation of the possible effect of a specified variation in the operational conditions on this efficiency.

The two essential operating characteristics are, for specified fluids and given θ_c and λ_c , the ratio $\mu = m'/m$ of the volumes of the driving and the entrained fluids, and the pressure $p_1 = \lambda_1 p_a$ in the mixer. It is a comparatively simple matter to analyze the influence of the operational conditions μ and λ_1 in the simple case where the two fluids have sensibly constant and equal specific heats. Such is the case in first approximation of air and (burnt) gases arising from the combustion of liquid fuel with a sufficient excess of air.

In that event, $r = h = h_d = 1$ and equations (16) to (24) are considerably simplified.

This simplification was utilized in the present report for the summary study of the operation of rockets with thrust augments tubes.

Translated by J. Vanier
National Advisory Committee
for Aeronautics

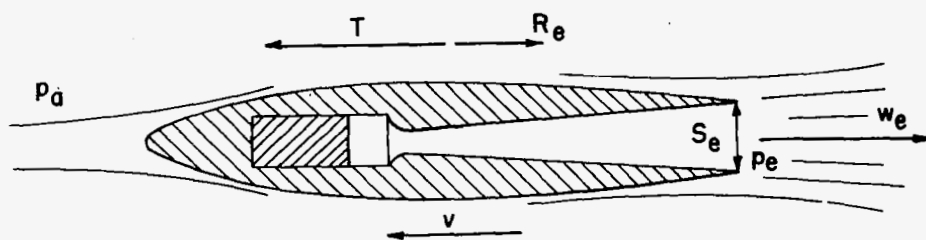
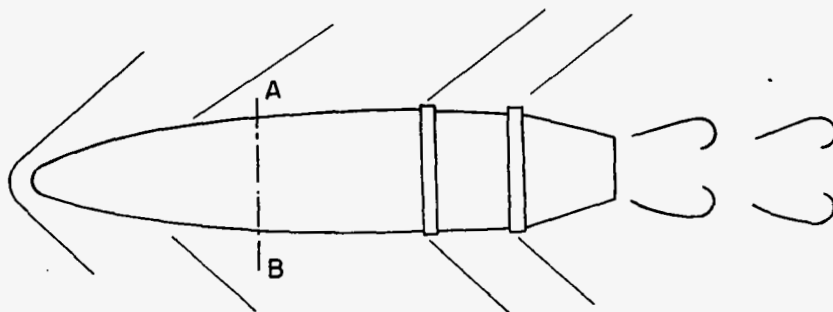
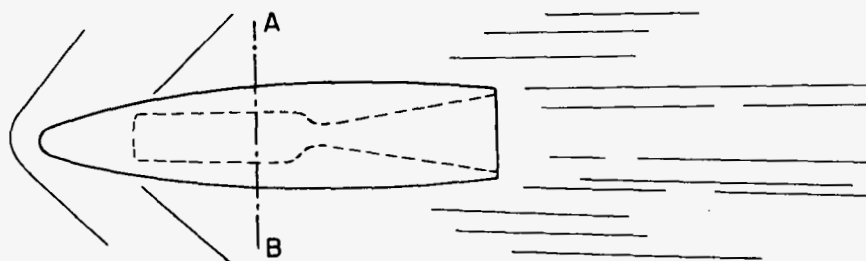


Figure 1.



High-speed shell



High-speed rocket

Figure 2.

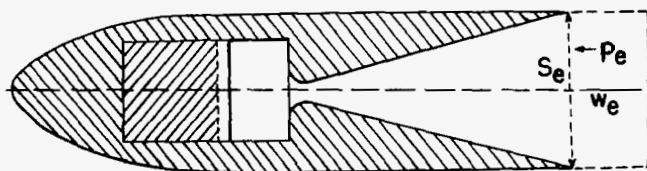


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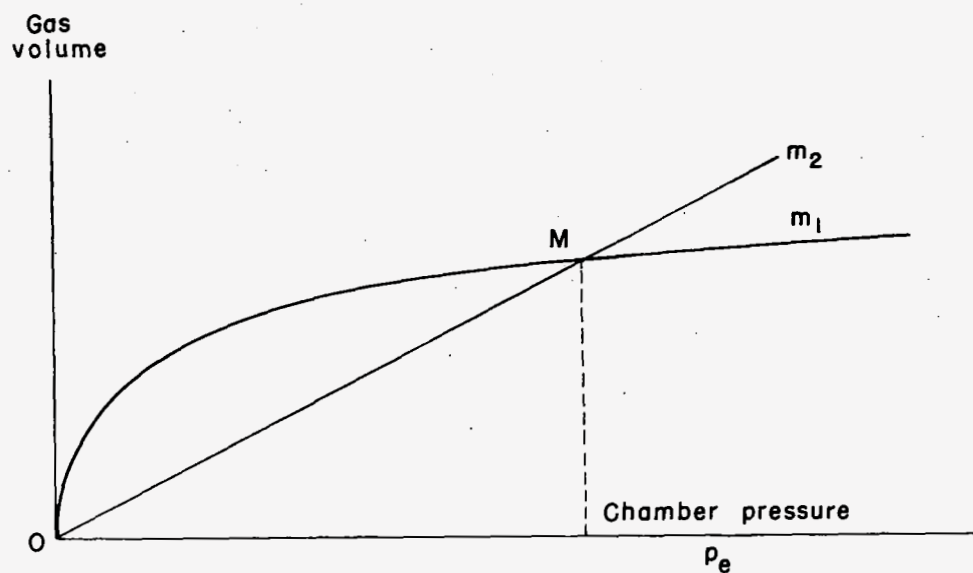


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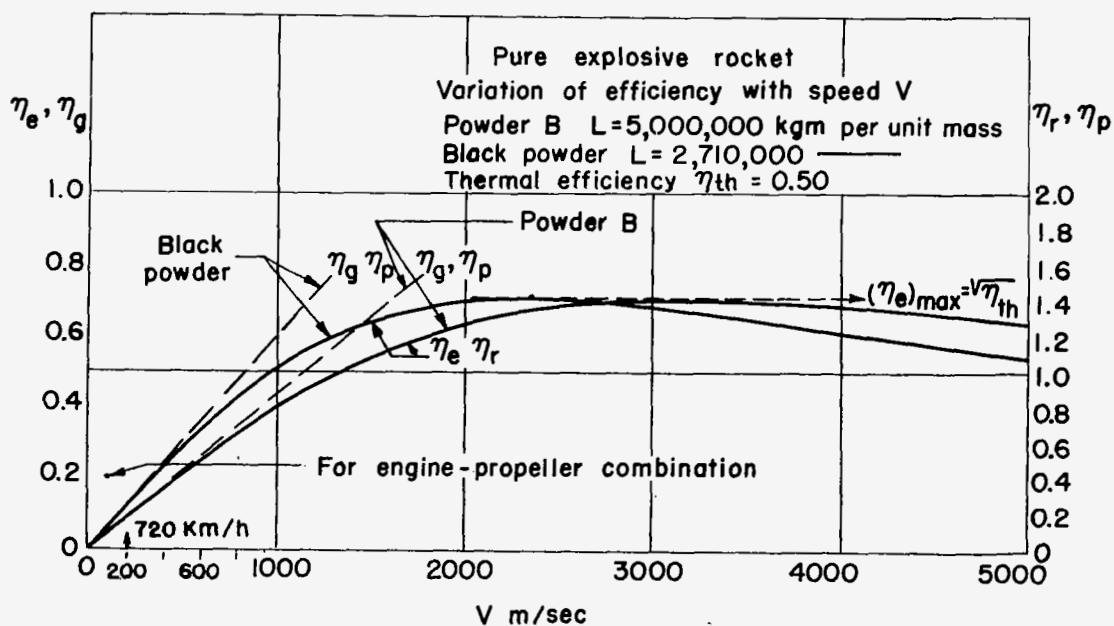


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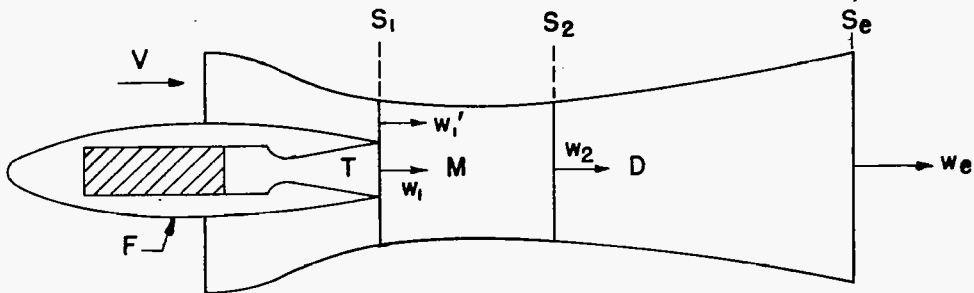


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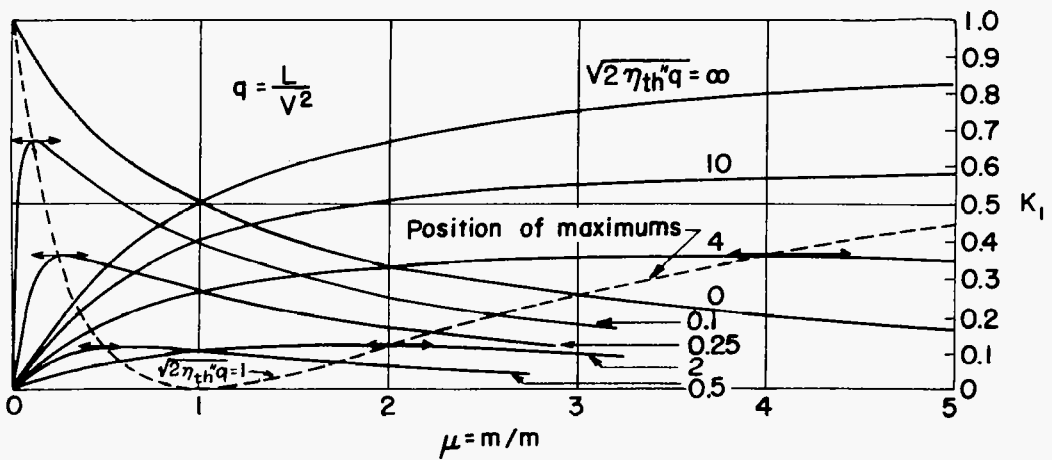


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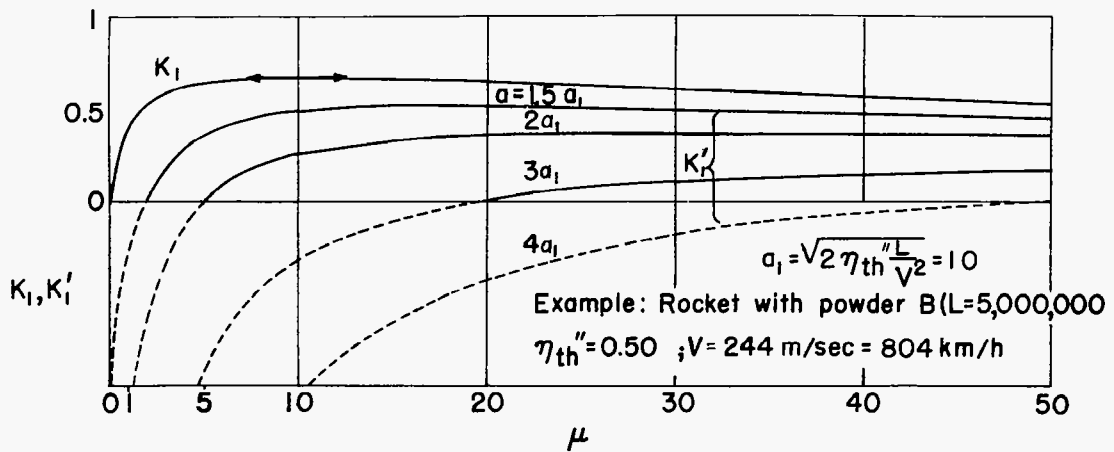


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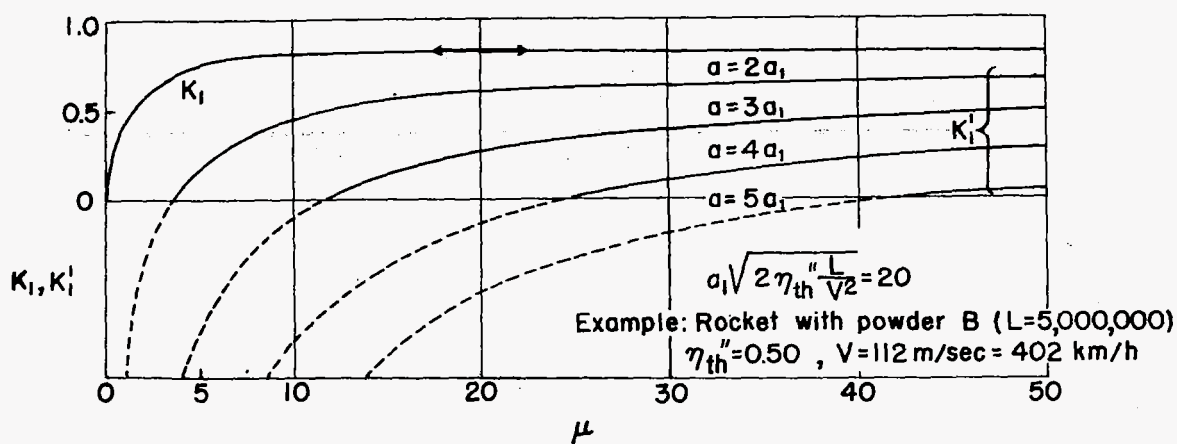


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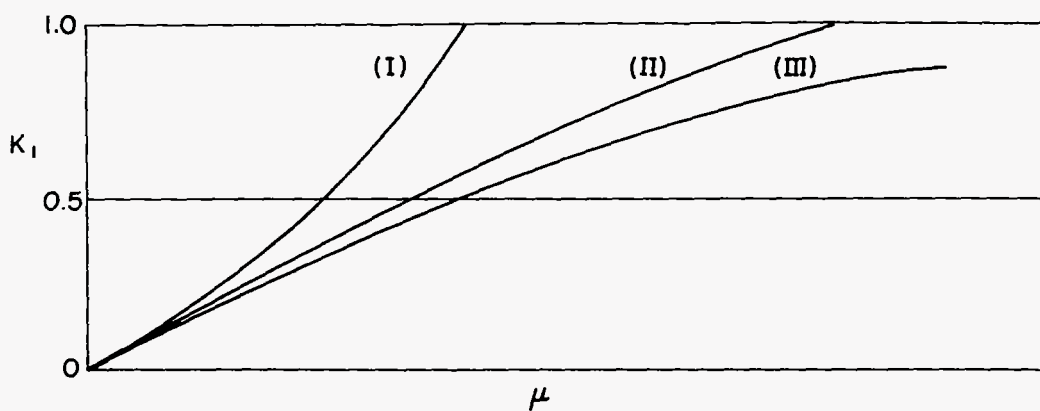


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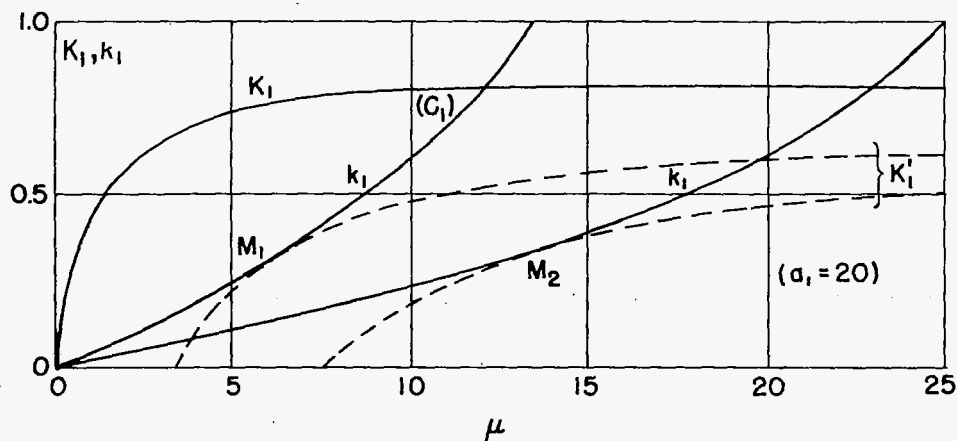


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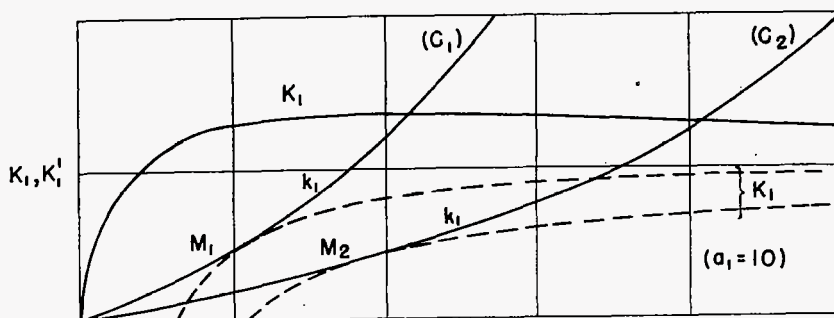


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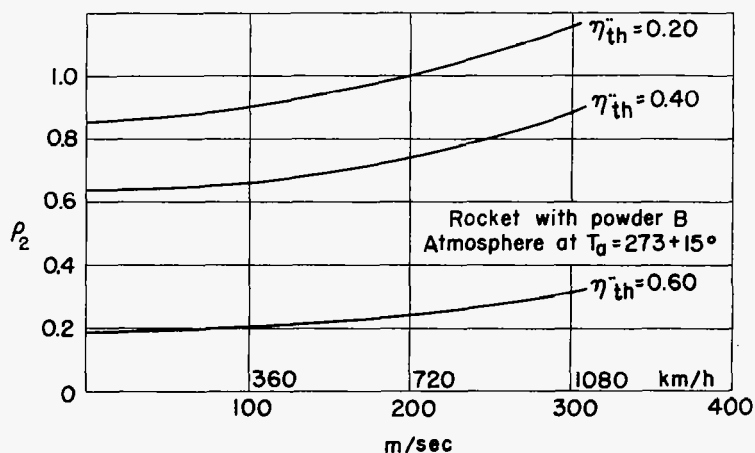


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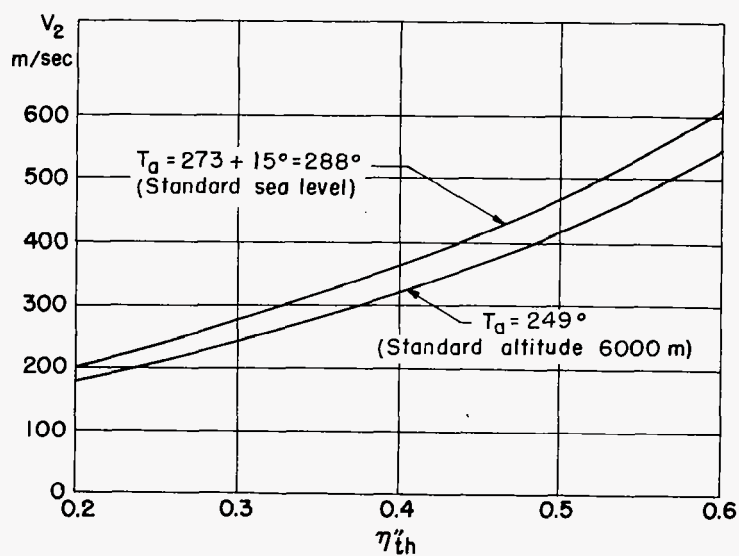


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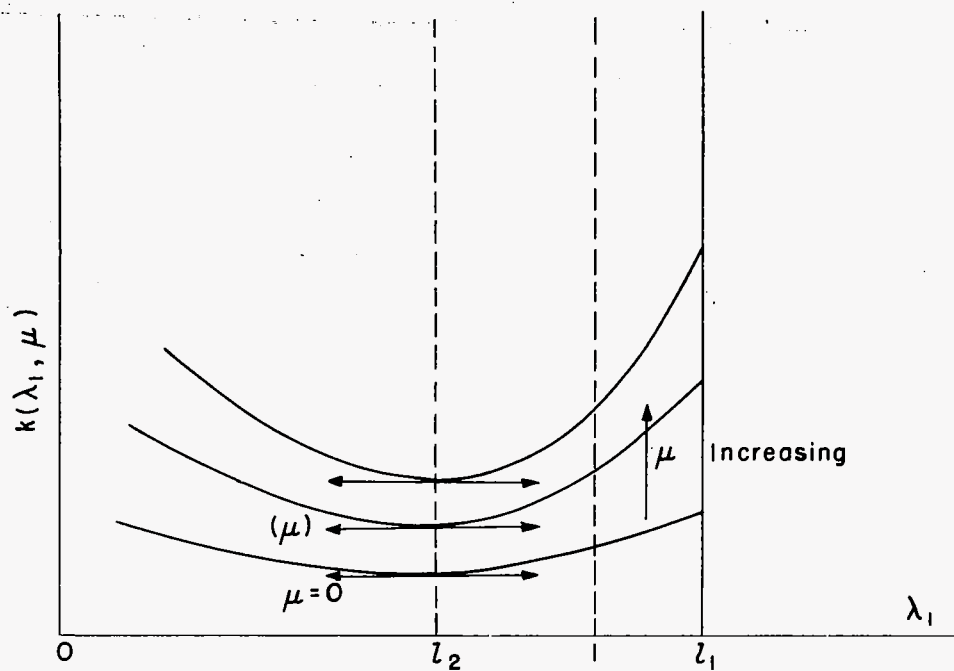


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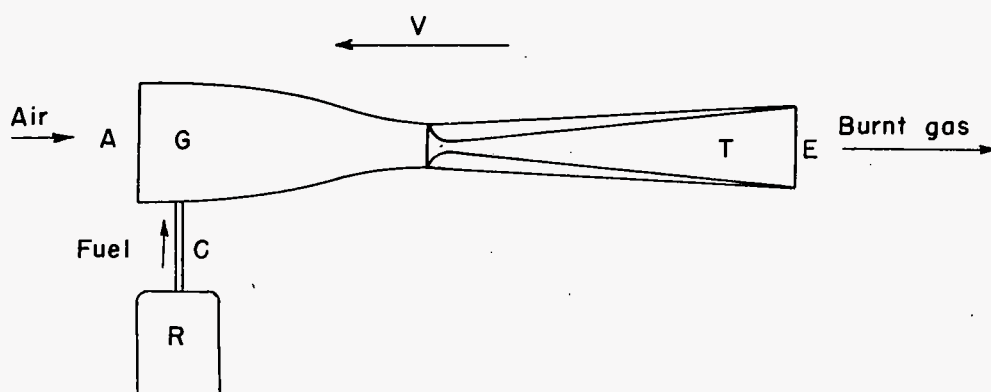


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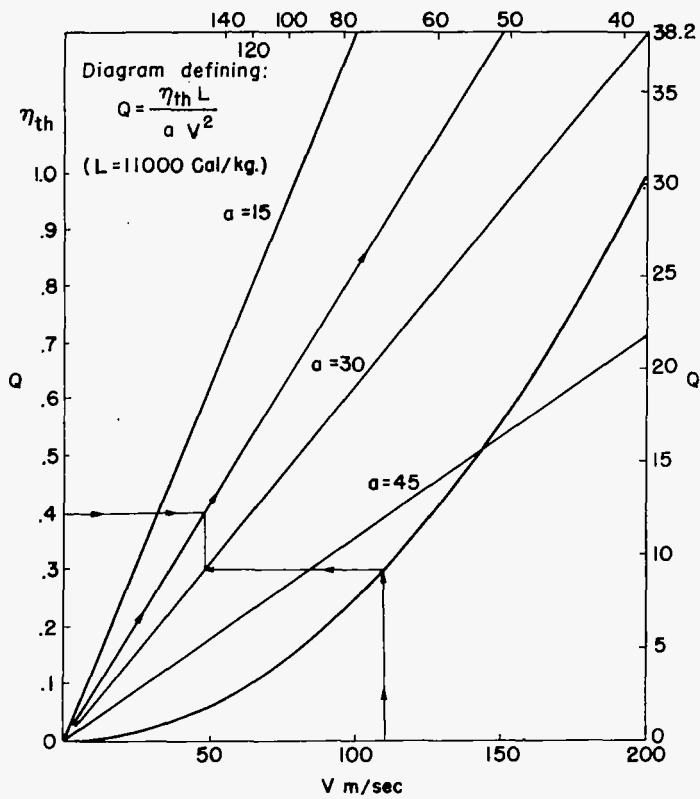


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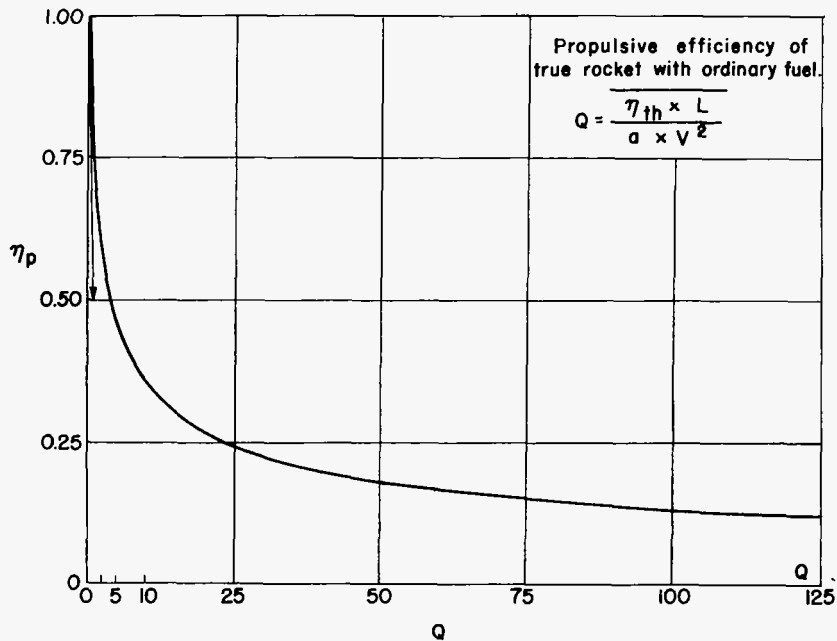


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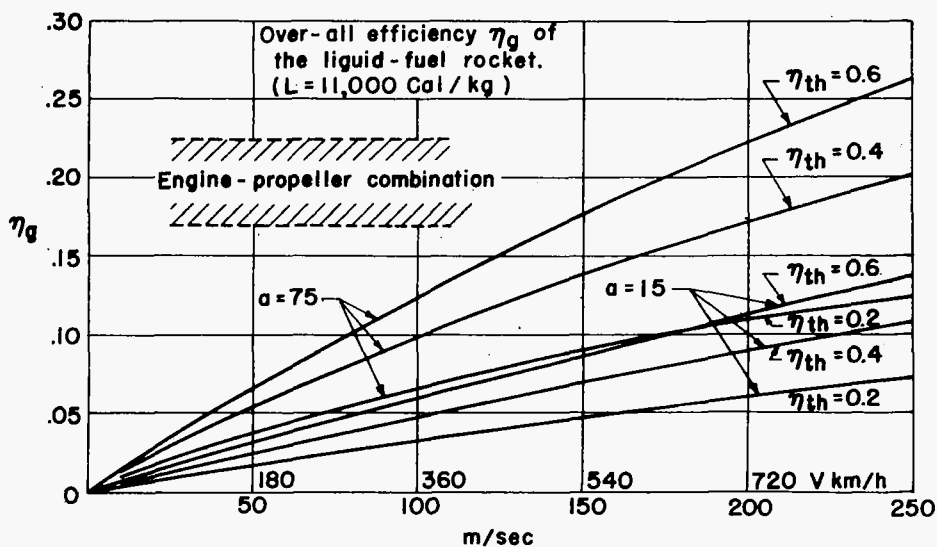


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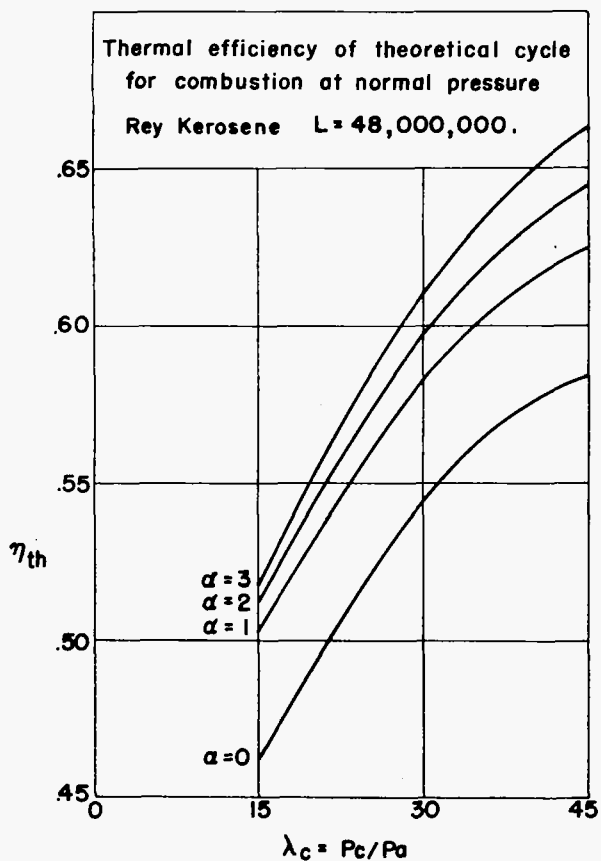


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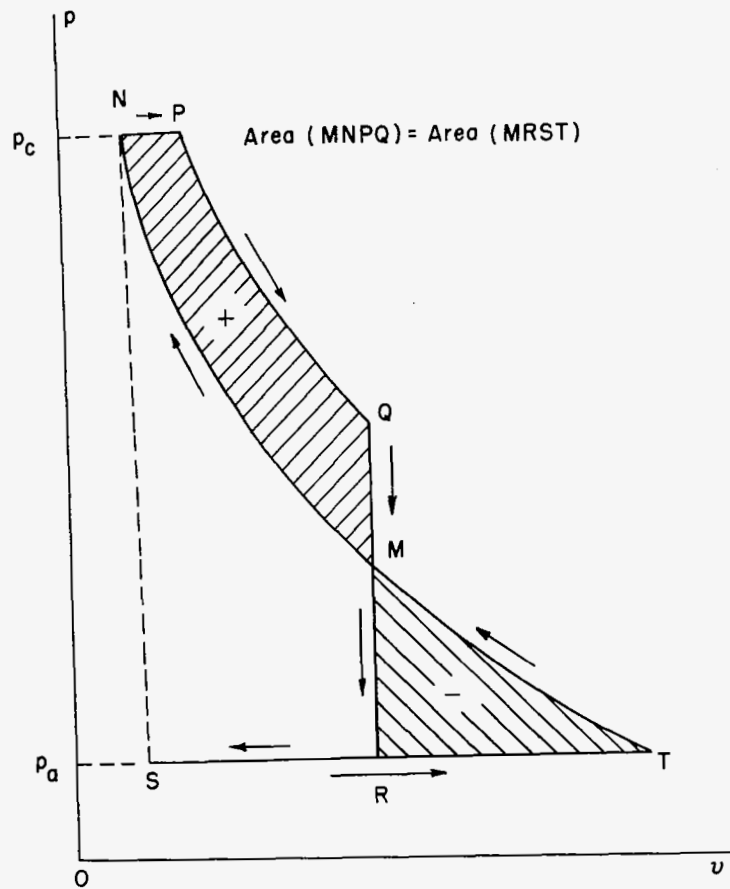


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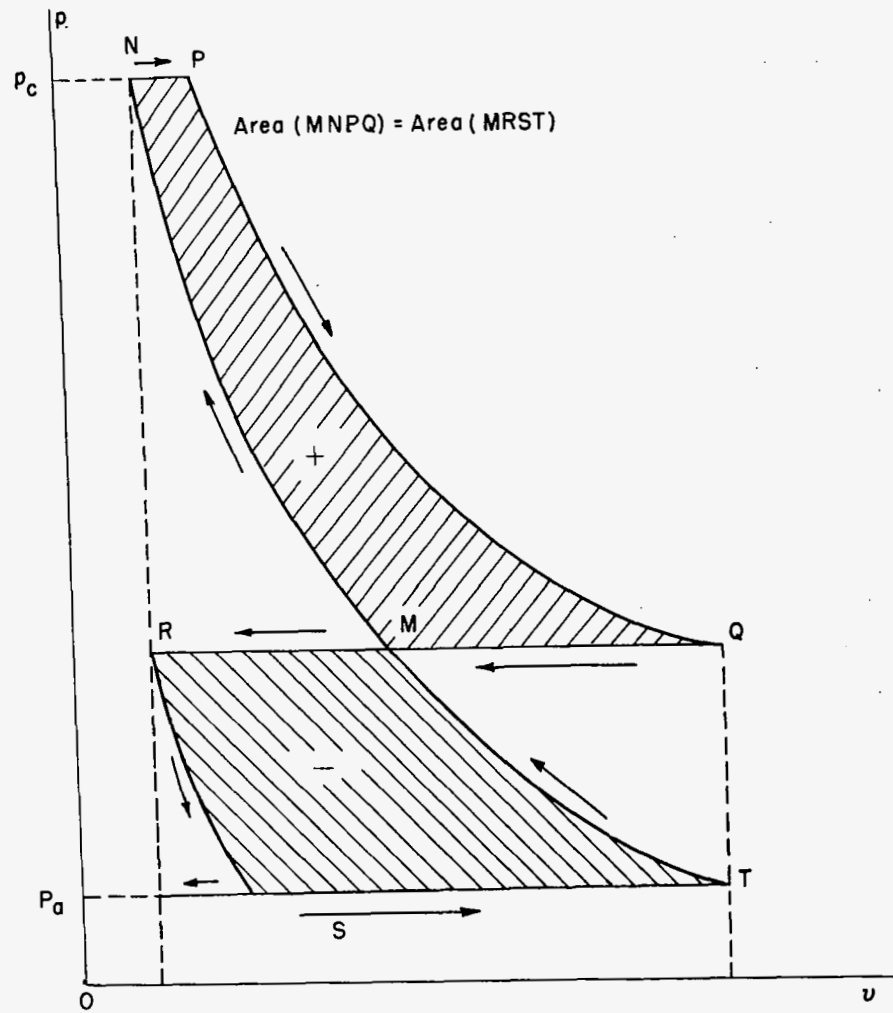


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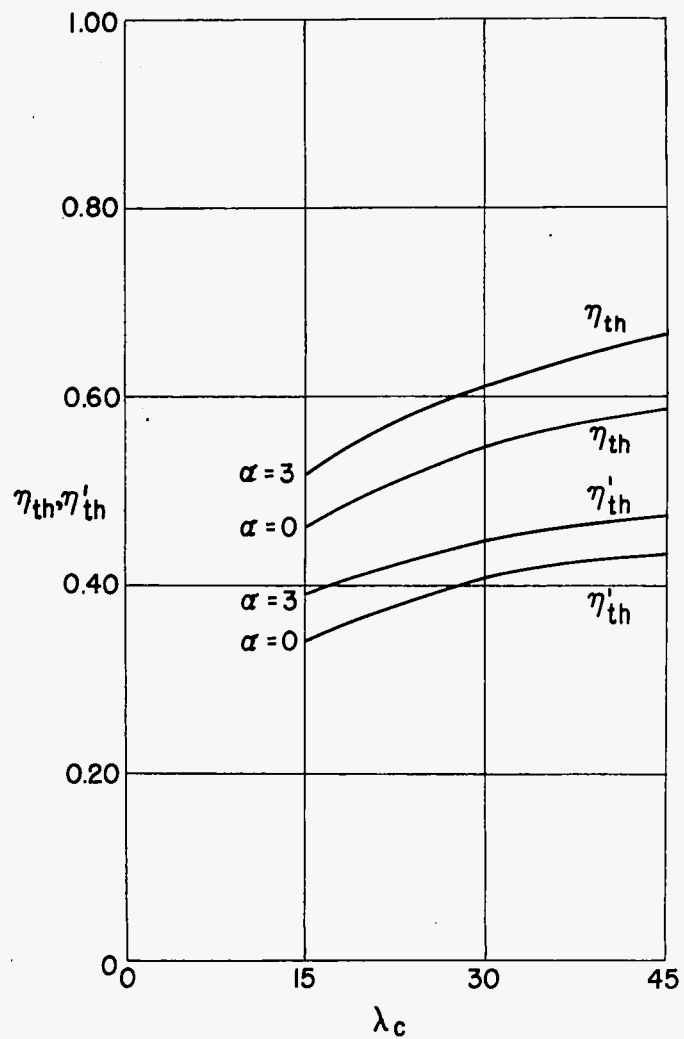


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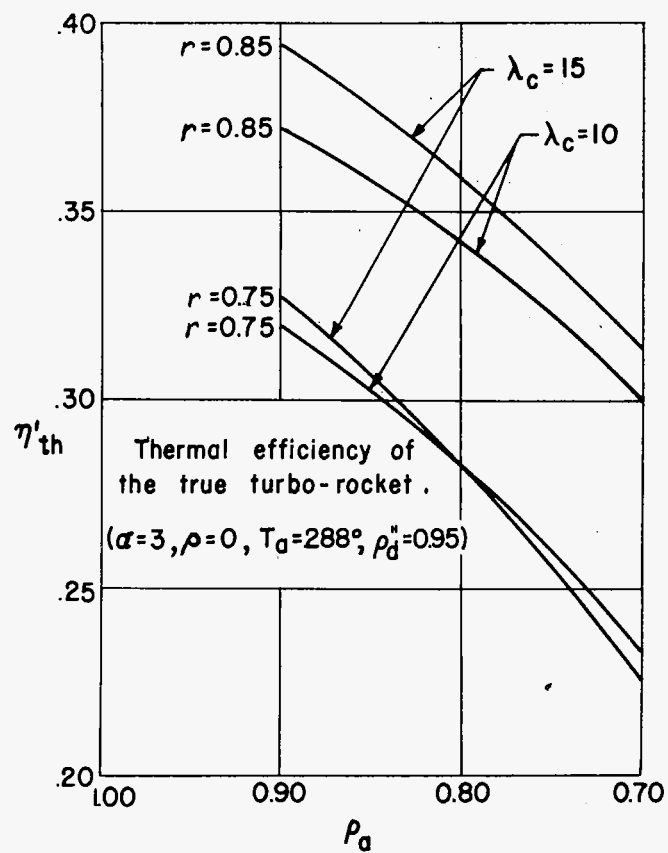


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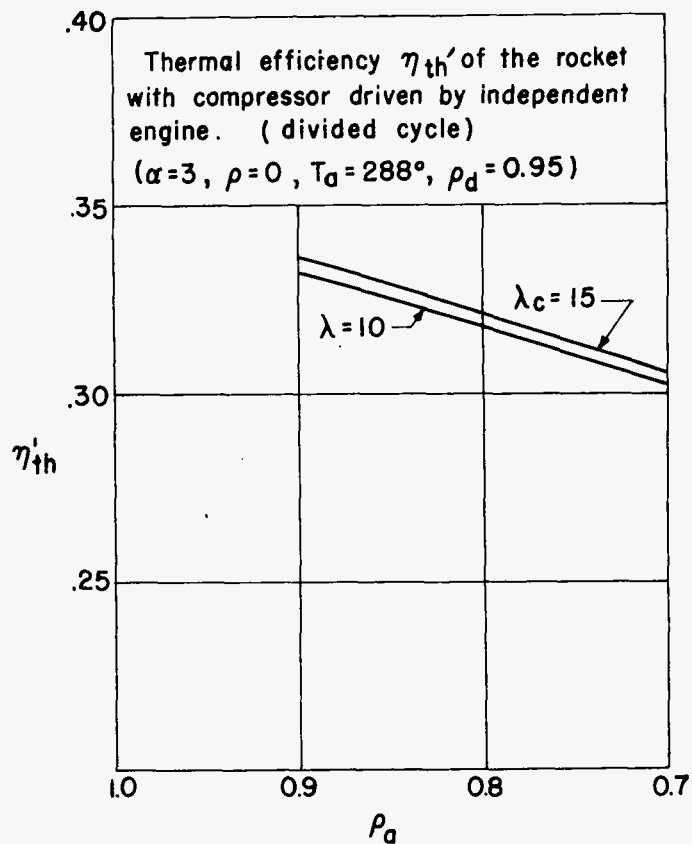


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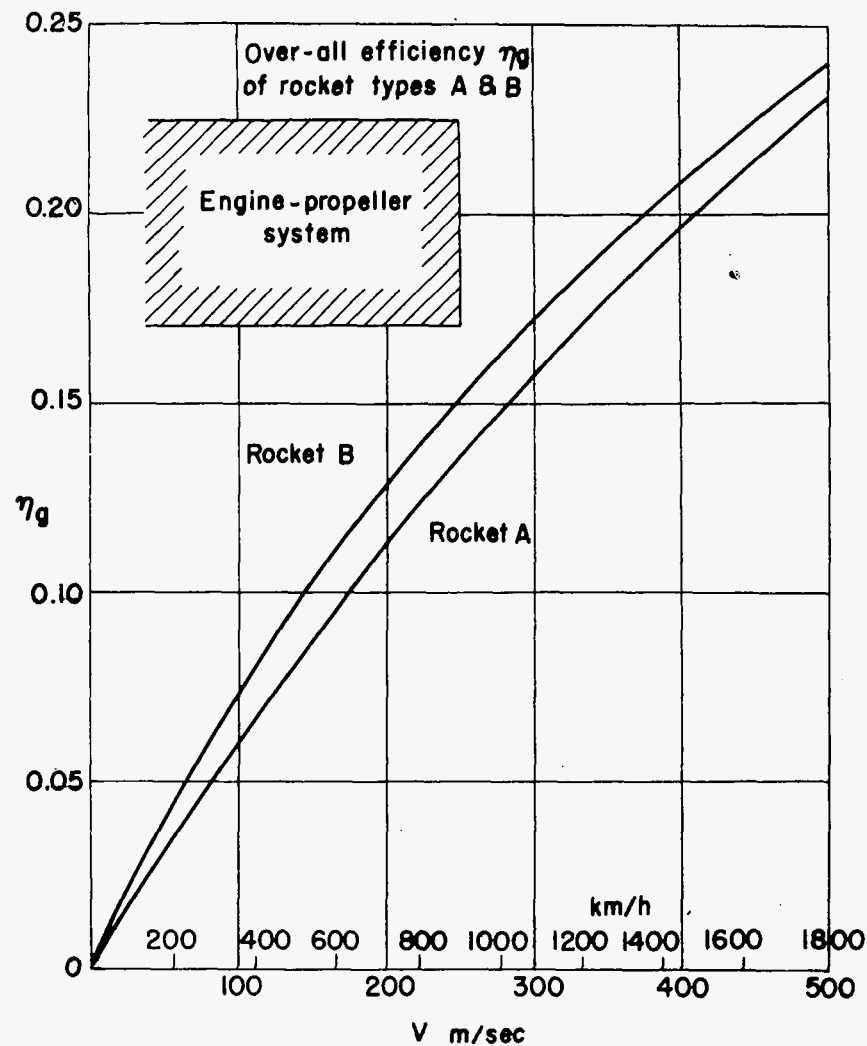


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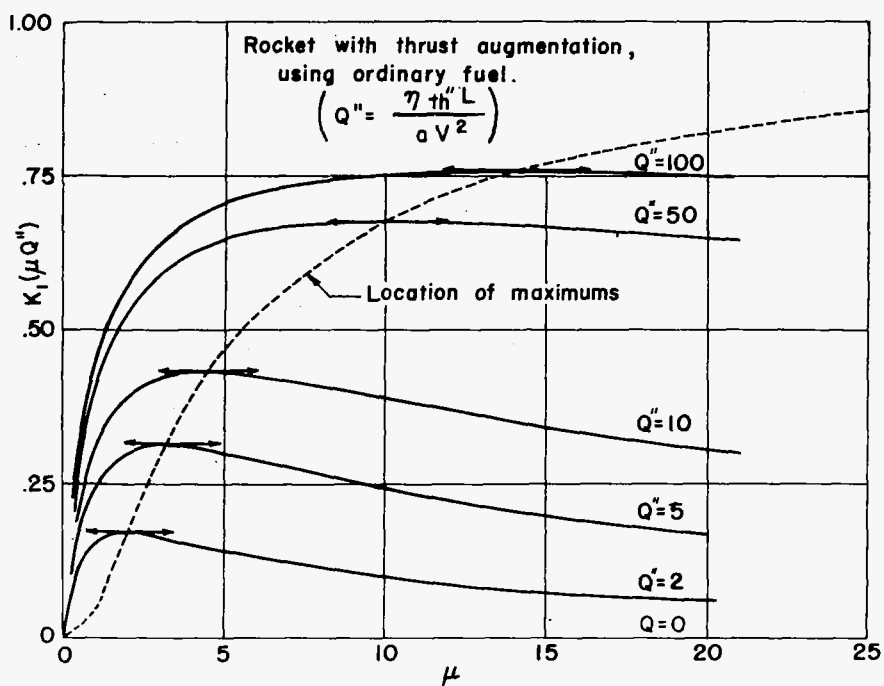


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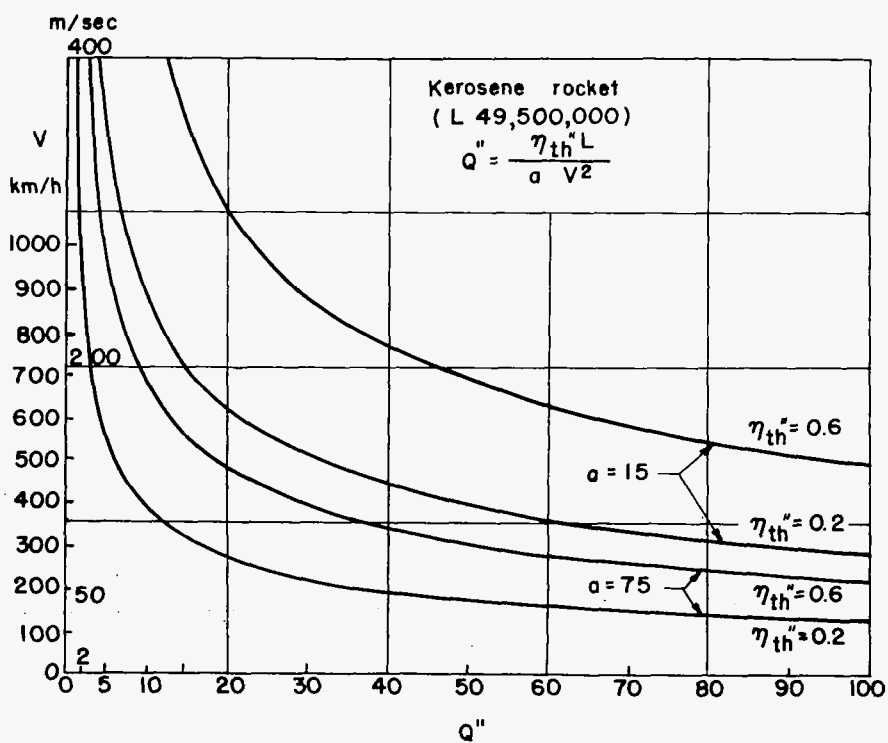


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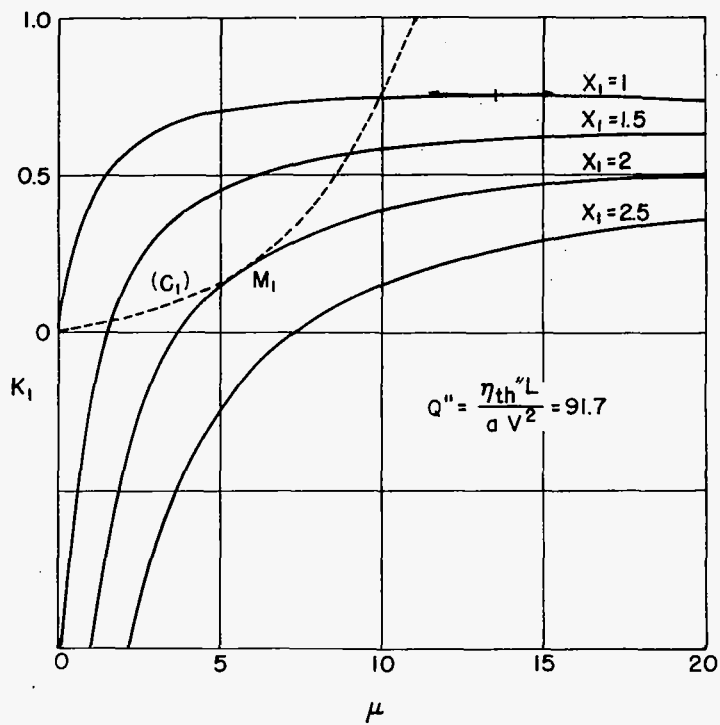


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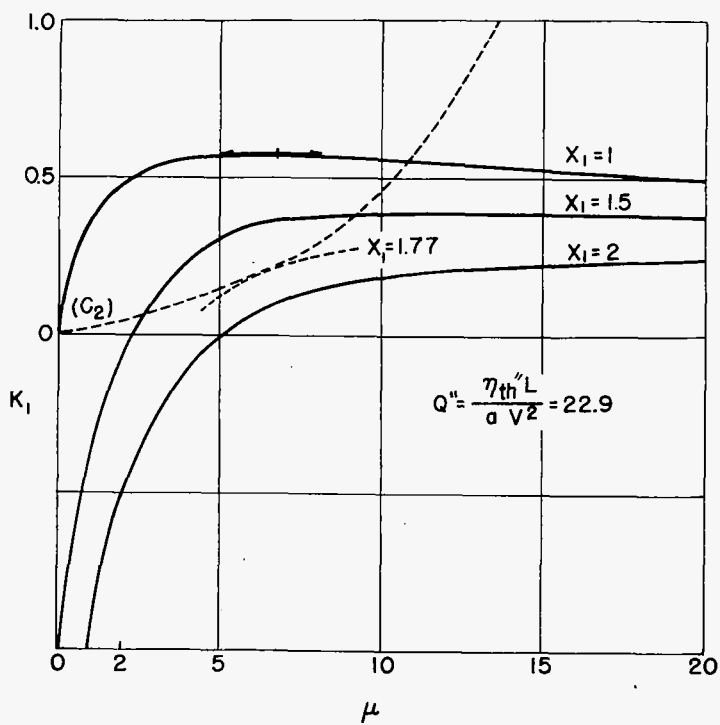


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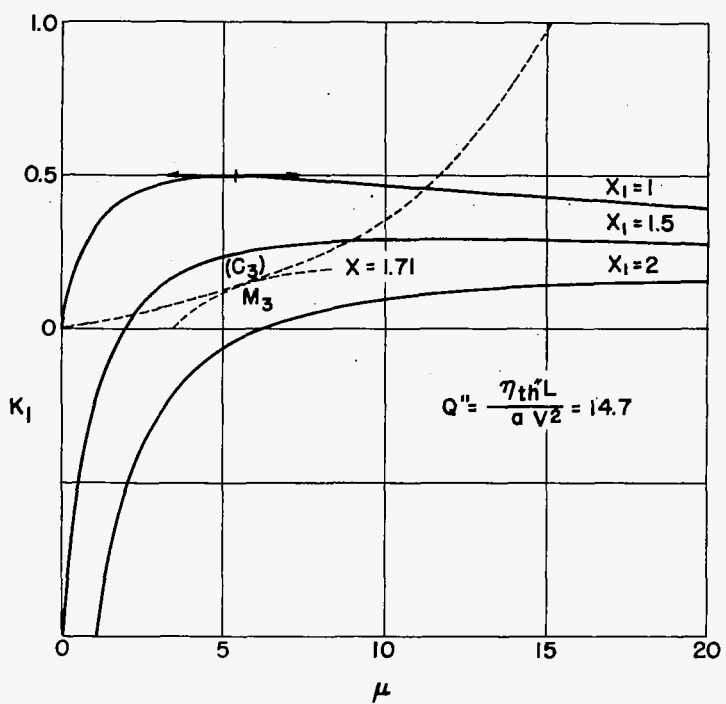


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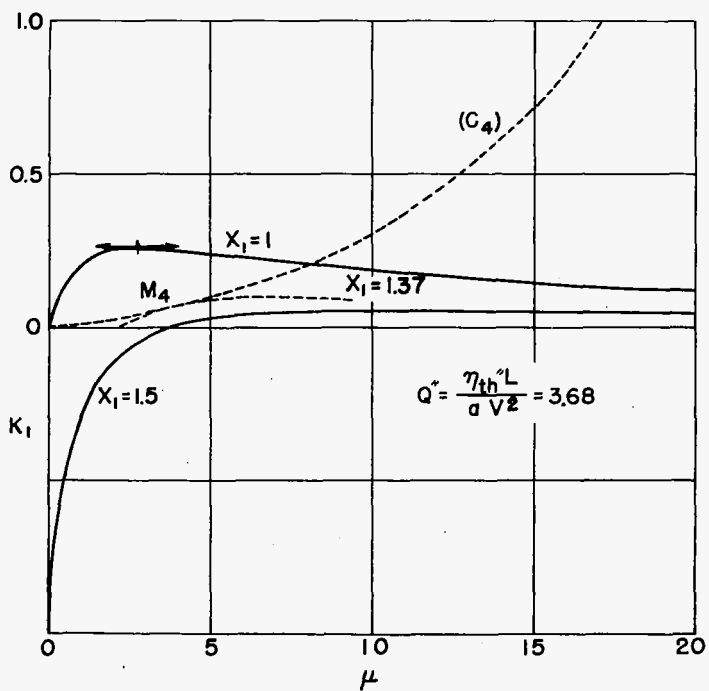


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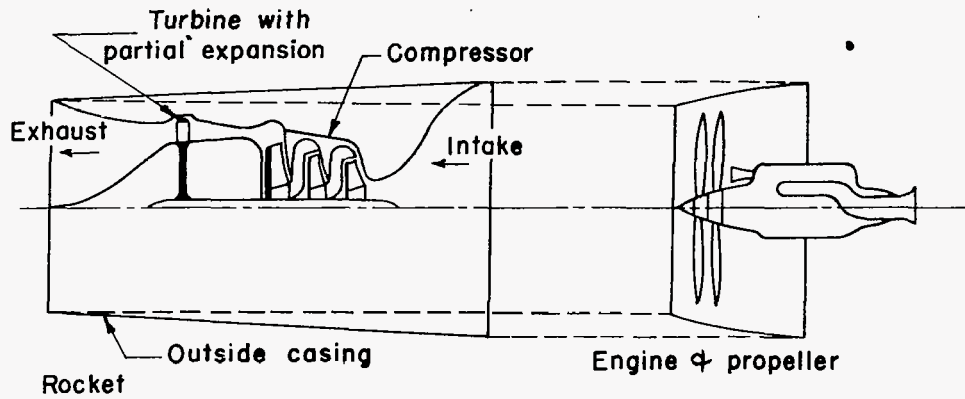


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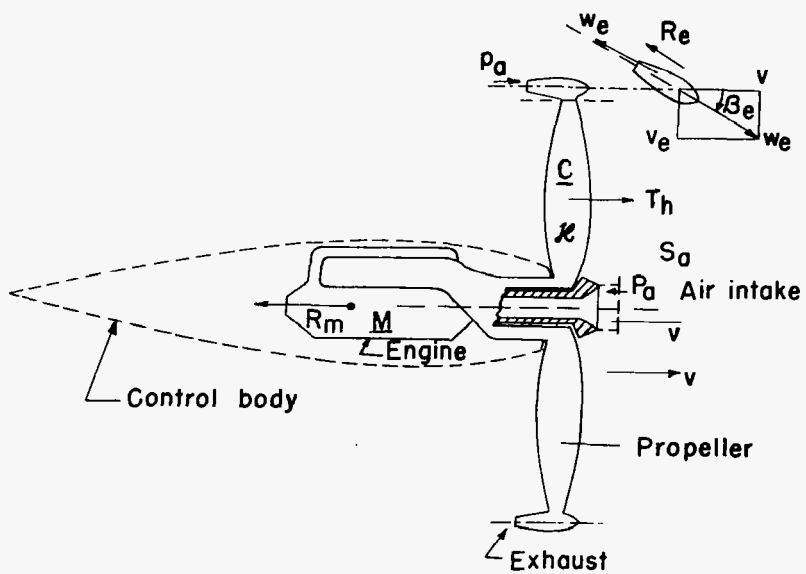


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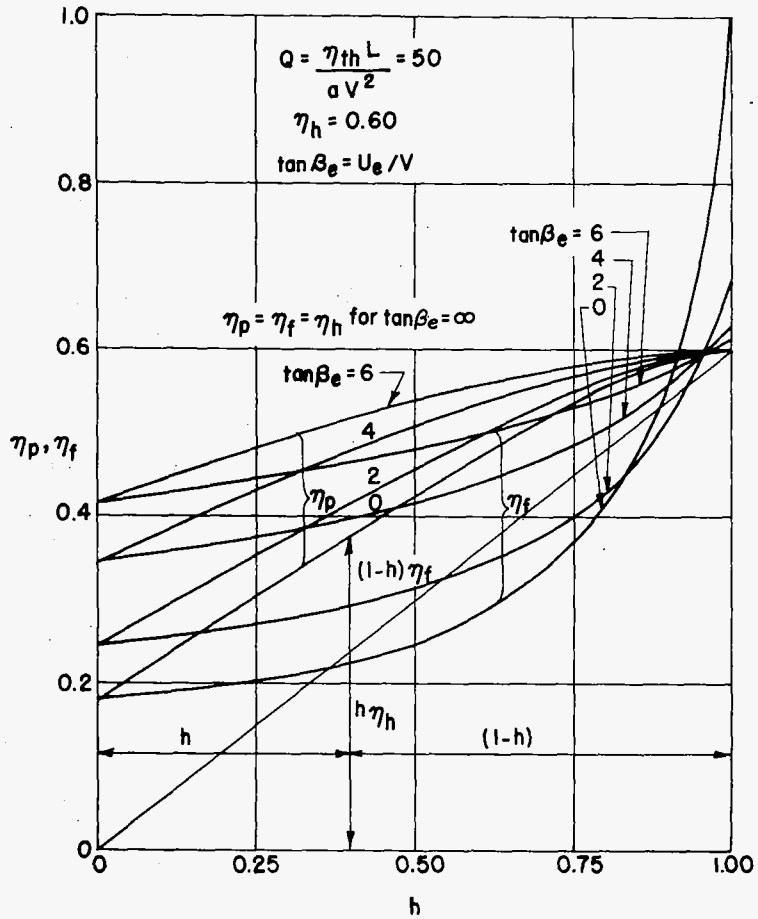


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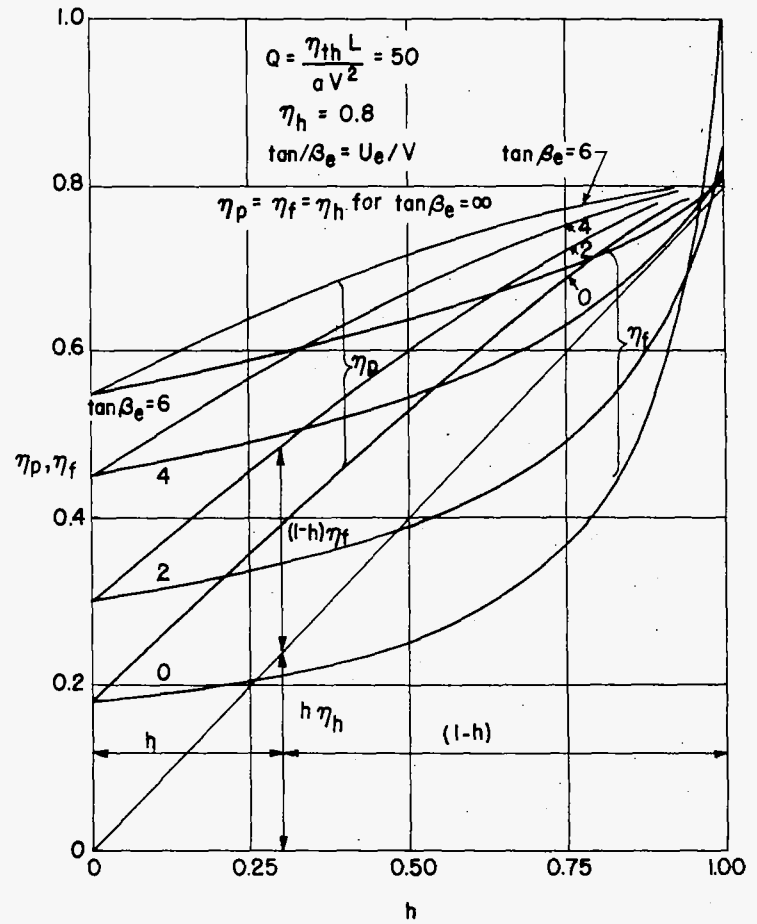


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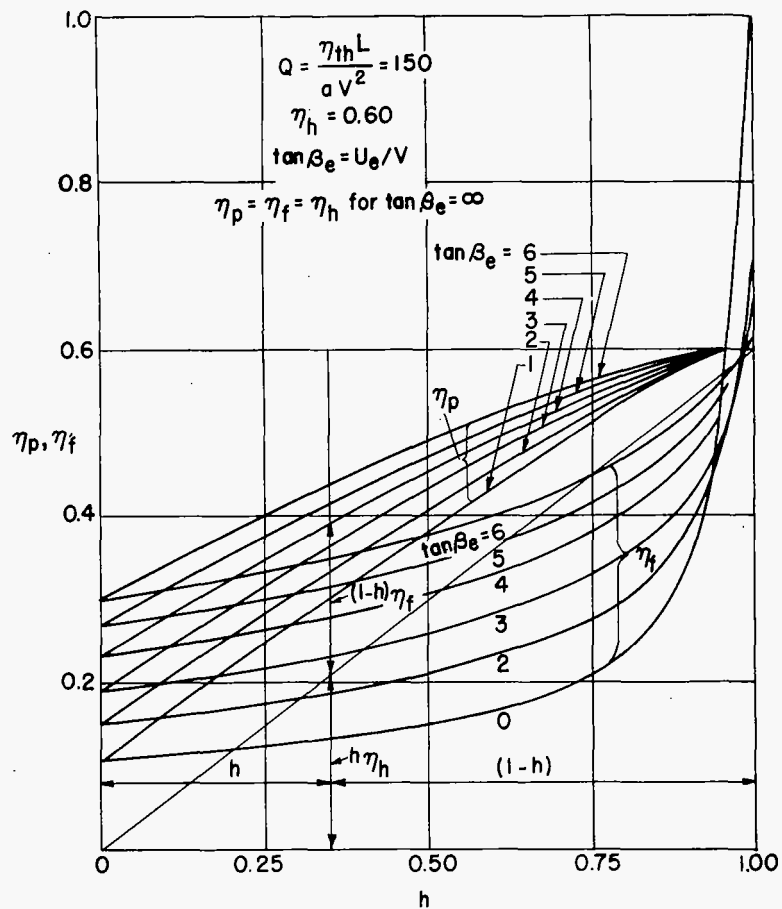


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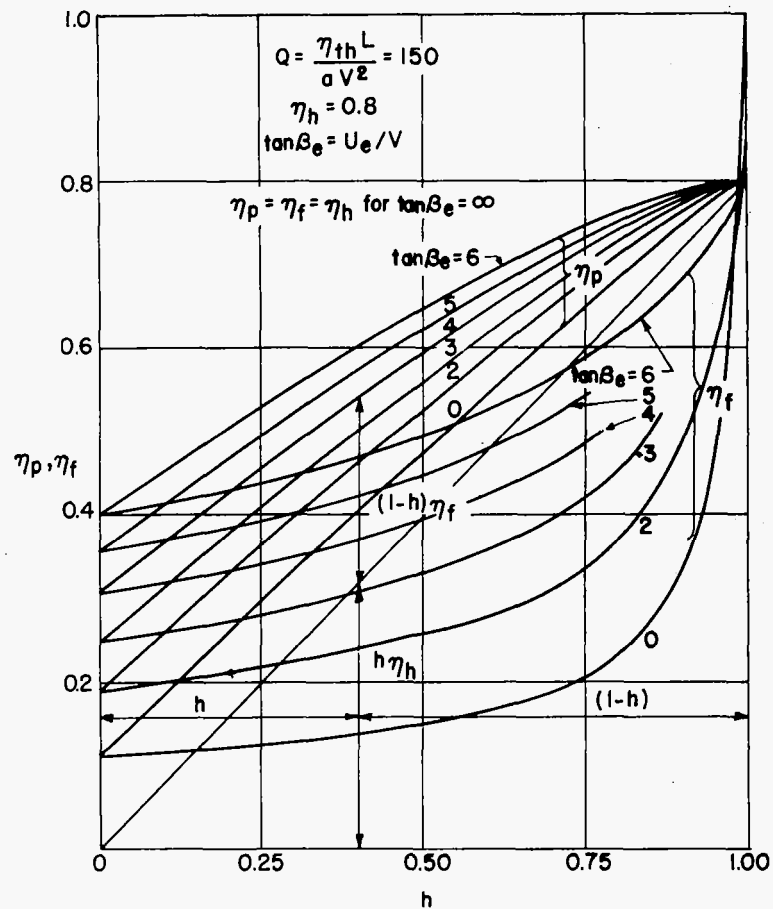


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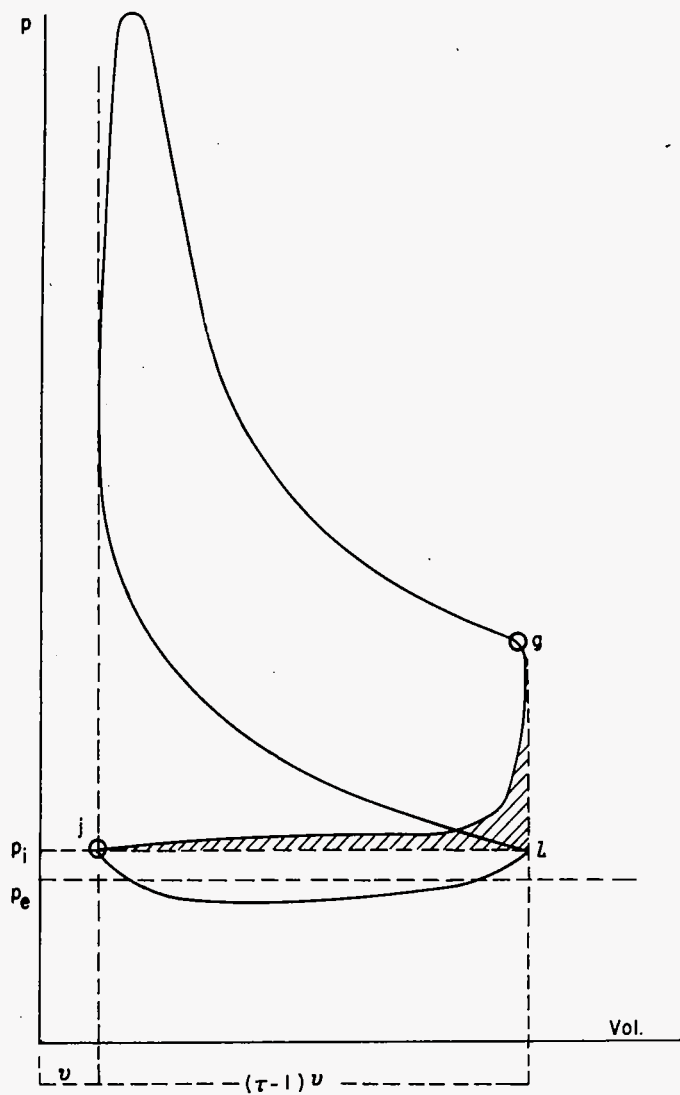


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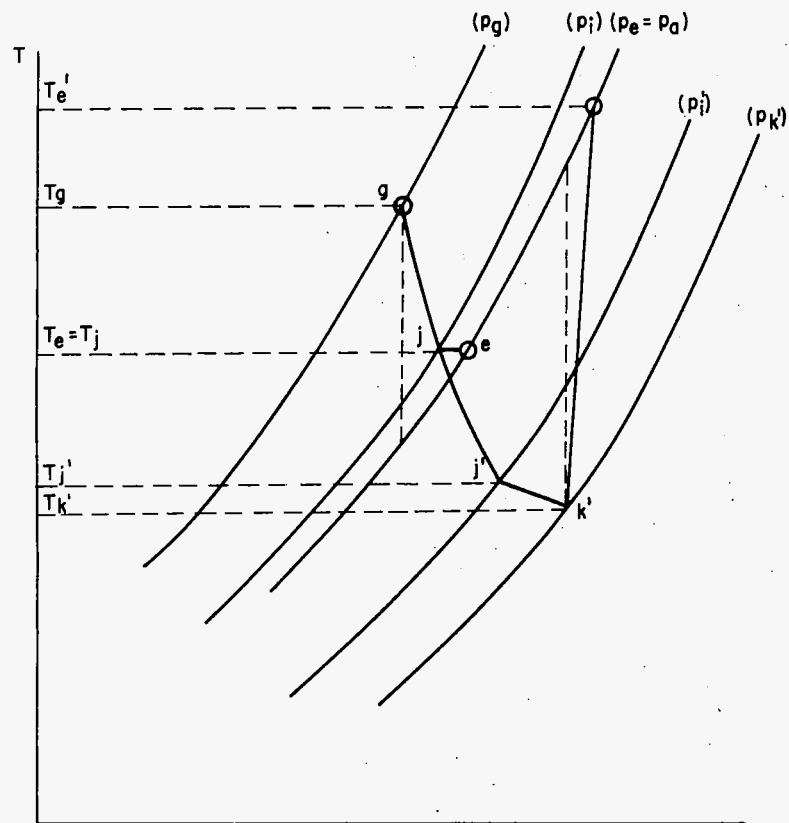


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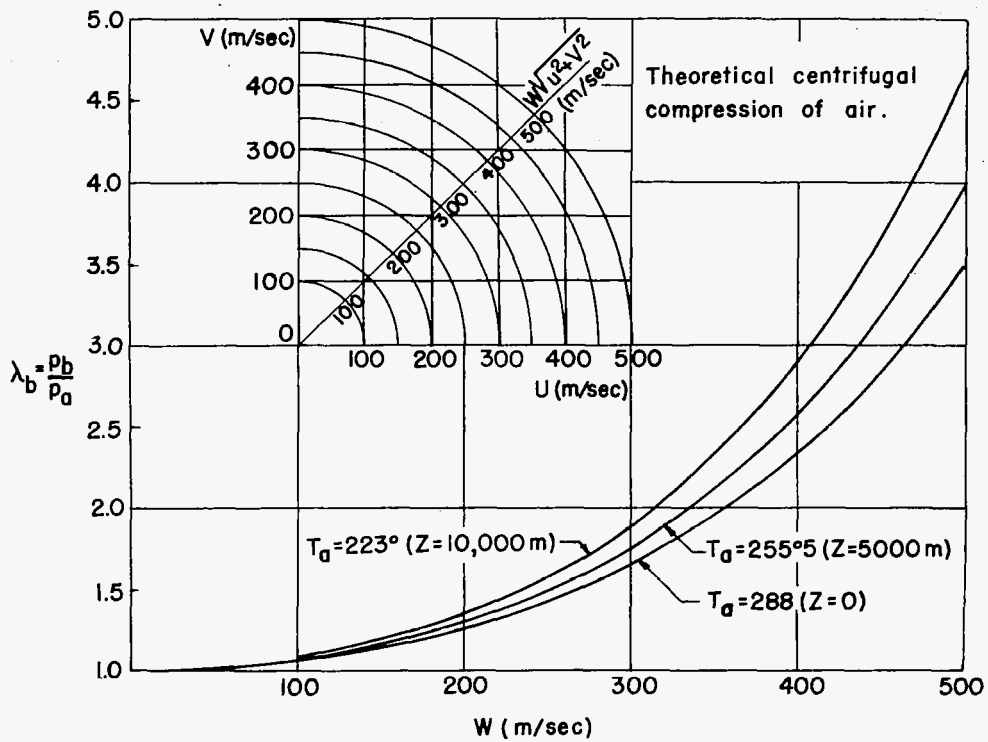


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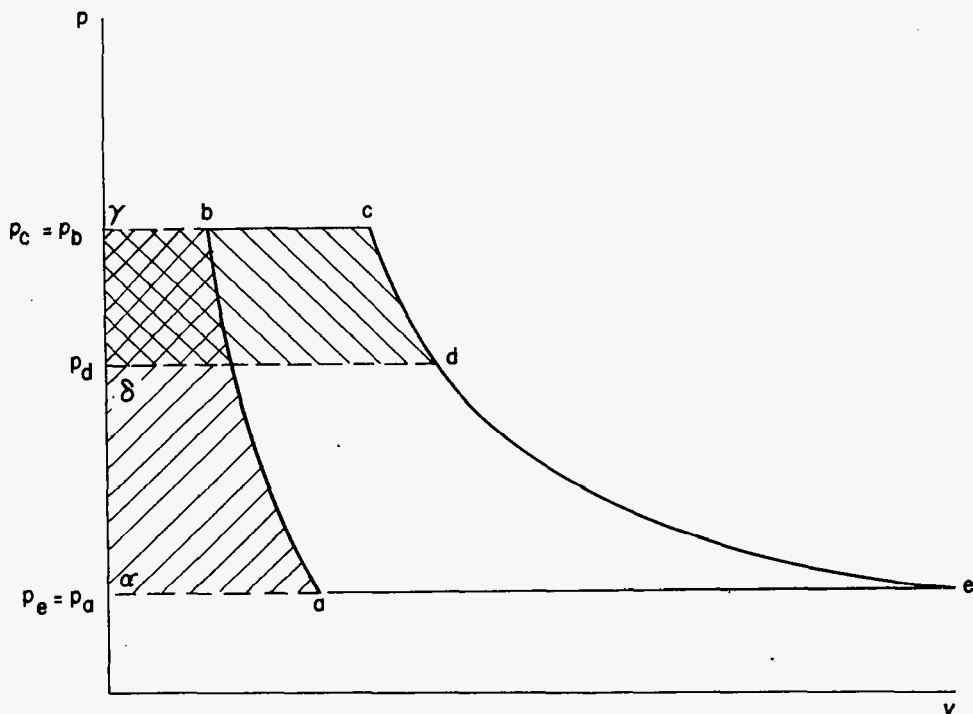


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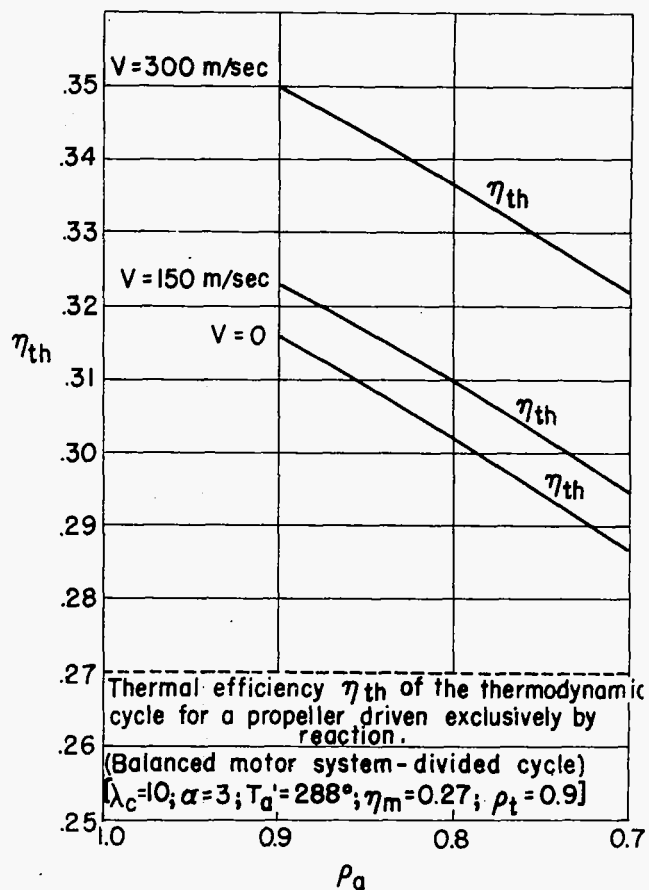


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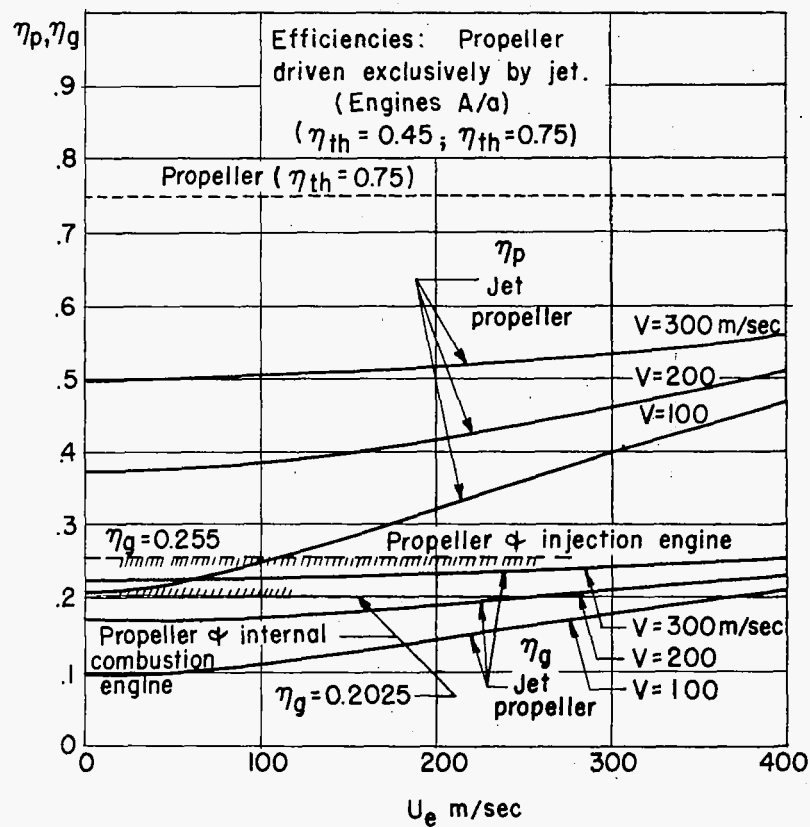


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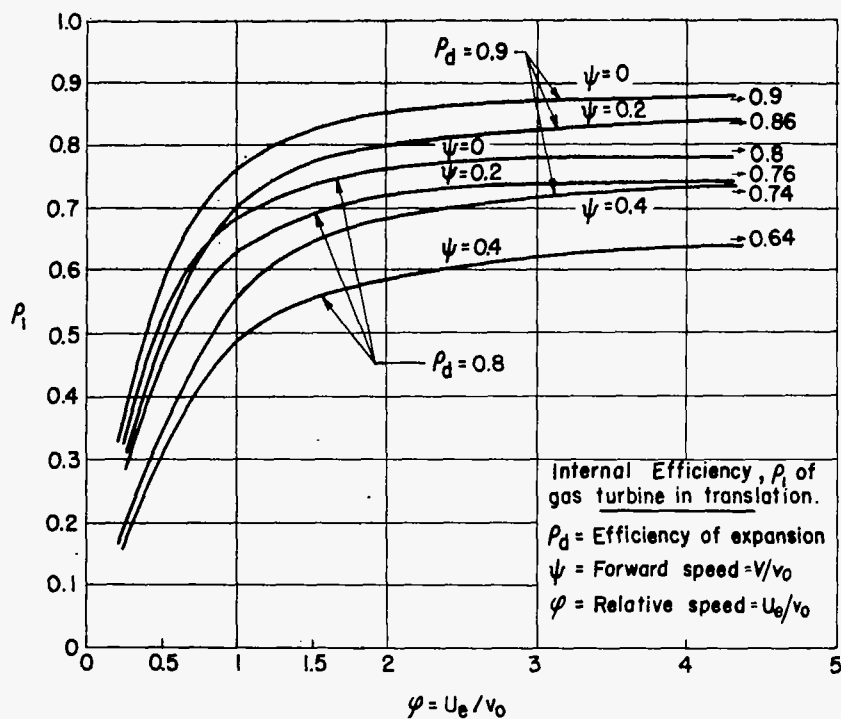


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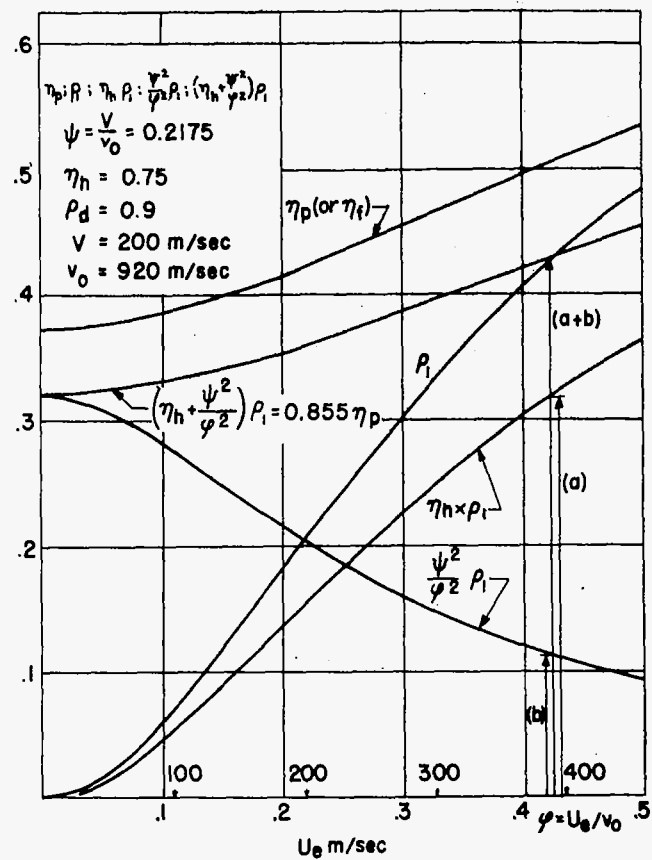


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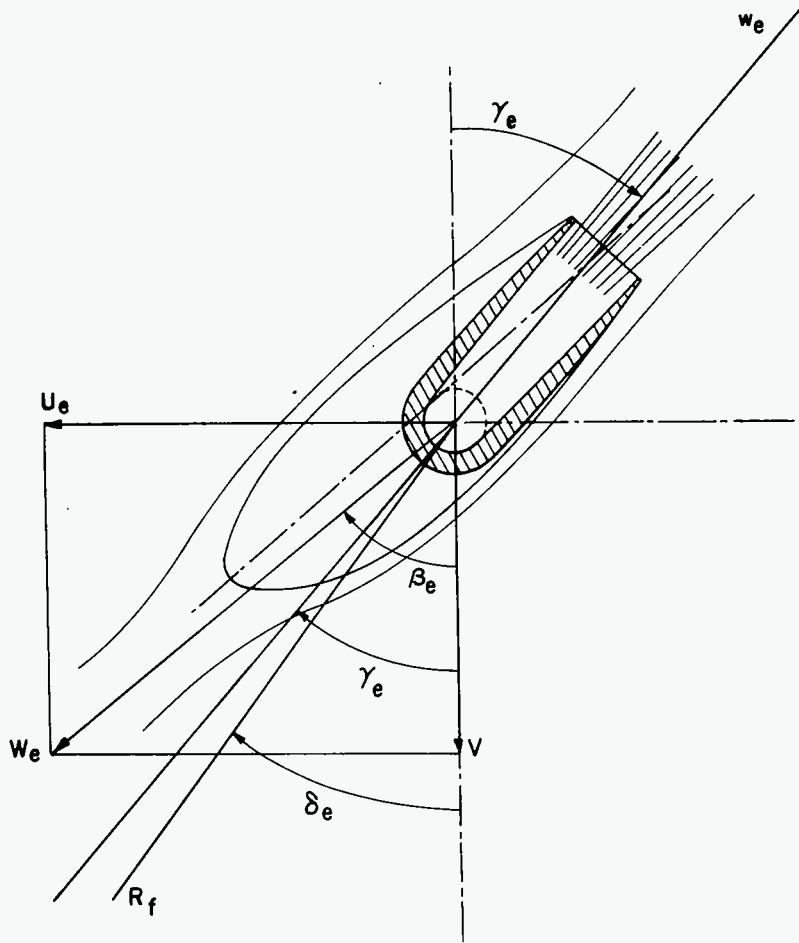


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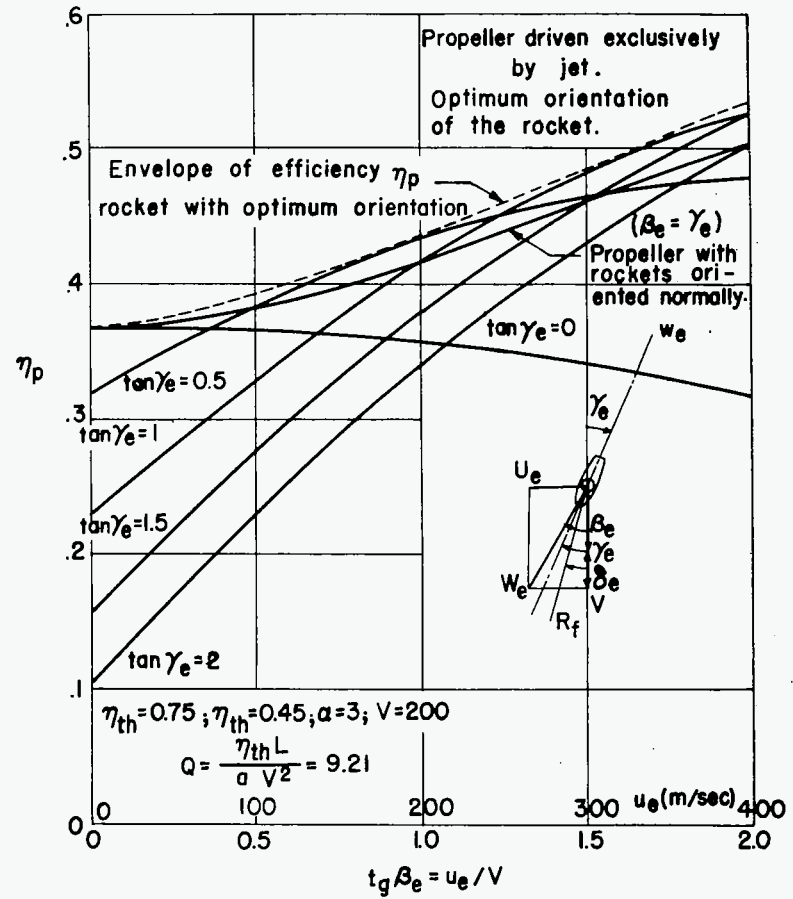


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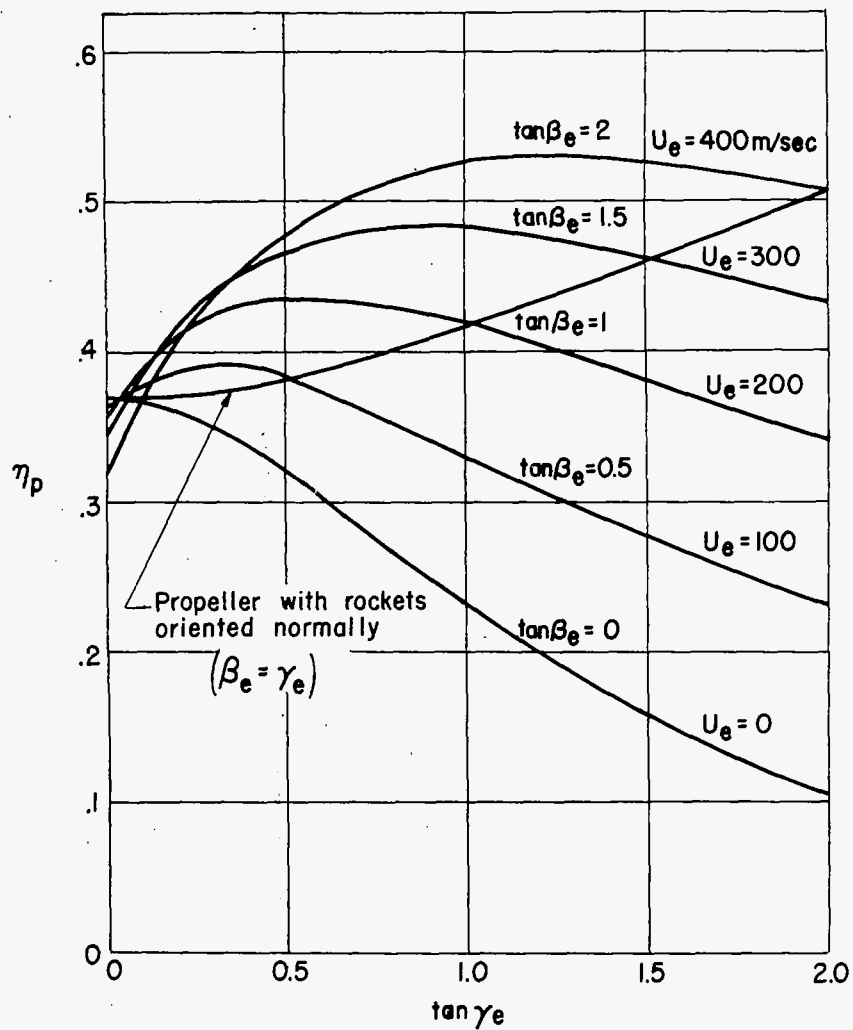


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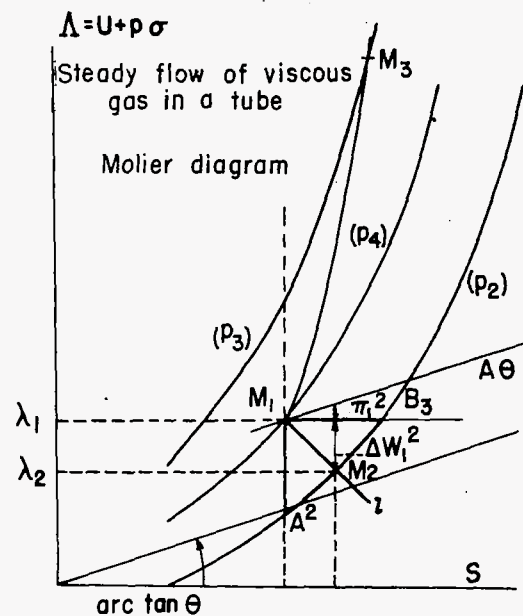


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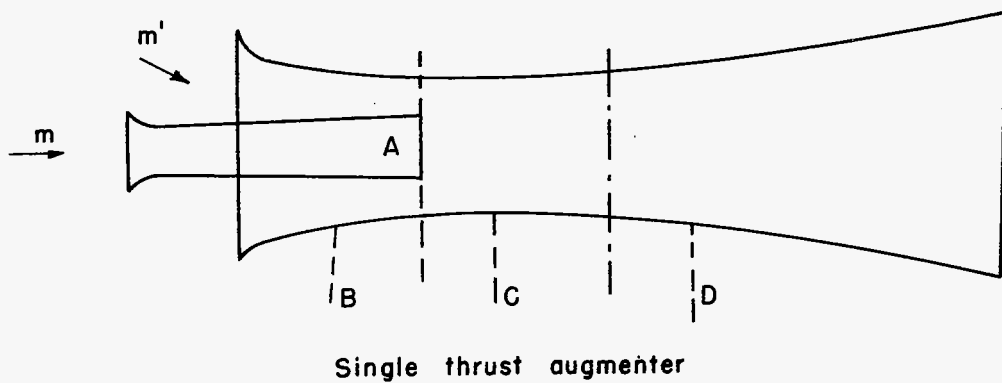


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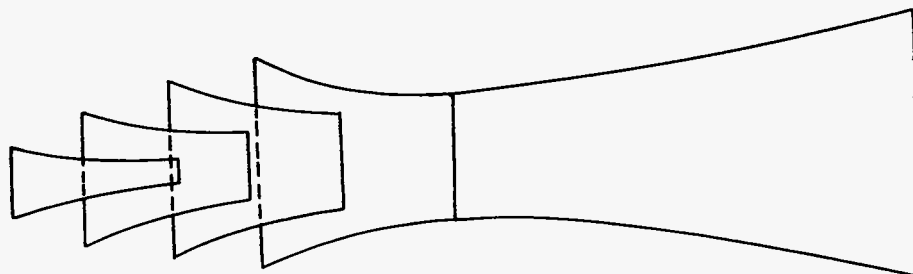


Figure 53.



Induced